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**THE STUDY OF DYNAMIC STALL AND URANS
CAPABILITIES ON MODELLING PITCHING AIRFOIL FLOWS**



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MODELLING AIRFOIL PITCHING FLOWS**

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Projeto de conclusão de curso **APROVADO** pelo
Colegiado do Curso de Graduação em Engenharia
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To mom and dad.

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*“Flying is learning how to throw yourself at
the ground and miss.”*

Douglas Adams – Life, the Universe and Everything

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ABSTRACT

This document describes the investigation of the behavior of the flow over a pitching NACA 0012 airfoil at Reynolds number $Re=100,000$ and $Re=2,500,000$ of the analysis of a two-dimensional $k-\omega$ SST (Shear Stress Transport) simulation. The behavior of the flow wake at the trailing edge is studied by the analysis of streamlines for each incidence angle and results are compared by the study of theoretical concepts and experimental data. The use of standard Unsteady Reynolds-Averaged Navier-Stokes (URANS) simulation has shown accuracy in predicting stall and reattachment incidence angles for upstroke and downstroke. The simulations were also capable of capturing flow information in agreement with experiments despite the over prediction of lift and drag coefficients, due to the two-dimensional simplification. The study has also compared the two-dimensional URANS $k-\omega$ SST turbulence model simulation data with previous results from three-dimensional simulation Wall Resolved Large Eddy Simulation (LES) and two-dimensional URANS $k-\epsilon$ Chien turbulence model analysis, showing that the qualitative behavior of the hysteresis loop is close for the computations, although there is a quantitative deviation in the results for upstroke and downstroke in the lift coefficient.

KEYWORDS: *dynamics stall, pitching airfoil, URANS, $k-\omega$ SST turbulence modeling, sliding meshes*

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RESUMO

O presente trabalho descreve a investigação do comportamento do escoamento sobre um aerofólio NACA 0012 em movimento de arfagem dinâmica em números de Reynolds $Re=100,000$ e $Re=2,500,000$ através da análise da simulação numérica bidimensional fazendo uso do modelo de turbulência $k-\omega$ SST (Shear Stress Transport). O comportamento da esteira do escoamento na região do bordo de fuga do aerofólio é estudado através da análise das linhas de corrente para cada ângulo de incidência durante o movimento de arfagem. Os resultados são comparados com estudos experimentais através de conceitos teóricos.

O uso das médias de Reynolds das Equações de Navier-Stokes, ou método URANS, mostrou acurácia na predição do estol dinâmico e dos ângulos de re-aderência da camada limite para movimento ascendente e descendente do aerofólio. As simulações também foram capazes de obter informações do escoamento em concordância com os resultados experimentais apesar da sobre-predição dos coeficientes de arrasto e sustentação, devido a abordagem bidimensional.

O estudo realizado também comparou os resultados da simulação bidimensional URANS do modelo de turbulência $k-\omega$ SST com análises encontradas em literatura para análise tridimensional via Simulação de Grandes Escalas (Wall Resolved LES) e também a simulação bidimensional URANS fazendo uso do modelo $k-\epsilon$ Chien, obtendo concordância qualitativa nos resultados dos três modelos, apesar de um visível desvio entre si para a análise do coeficiente de sustentação durante fases ascendente e descendente do movimento de arfagem.

PALAVRAS CHAVE: *estol dinâmico, arfagem dinâmica, URANS, modelo de turbulência $k-\omega$ SST, malhas deslizantes*

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List of Abbreviations and Acronyms

CFD	Computational Fluid Dynamics
DDES	Delayed Detached Eddy Simulation
ELDV	Embedded Laser Doppler Velocimetry
LES	Large Eddy Simulation
MFLab	Laboratory of Fluid Mechanics
NSE	Navier-Stokes Equations
OES	Organized Eddy Simulation
PIV	Particle Image Velocimetry
LDA	Laser Doppler Anemometer
LDV	Laser Doppler Velocimetry
LSV	Laser Sheet Visualization Method
RANS	Reynolds-Averaged Navier-Stokes
RNG	Renormalization Group Theory
SST	Shear Stress Transport
UDF	User Defined Function
URANS	Unsteady Reynolds-Averaged Navier-Stokes
TRANS	Transient Reynolds-Averaged Navier-Stokes

List of Symbols

α	Airfoil angle-of-attack	[rad]
A	Pitching axis position relative to chord from leading edge	[-]
b	Half chord (as conventional in Aeroelasticity), $b = c/2$	[m]
c	Airfoil chord	[m]
$\mathcal{C}(k)$	Theodorsen's Function	[-]
C_D	Drag coefficient	[-]
C_L	Lift coefficient	[-]
C_M	Momentum coefficient	[-]
C_P	Pressure coefficient	[-]
$\delta\bar{\alpha}$	Average pitch step	[rad]
ξ	Stream-wise coordinate	[-]
k	Pitch reduced frequency, $\omega c/2U_\infty$	[-]
$L(t)$	Lift Force	[N]
Re	Reynolds Number	[-]
U_∞	Free-stream flow velocity	[m/s]
T	Pitching Period	[s]
ω	Circular frequency of oscillation	[rad/s]

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1 Introduction

The flow past airfoils has been studied for almost a century when considering the Thin Airfoil Theory (Munk, 1922) applied to steady airfoils. These studies are the edge of aeronautical research, which allows the ongoing optimization of aeronautical profiles in modern aircraft. Unsteady effects used to be ignored for simplicity during experimental research; hence the flow over conventional fixed airfoils is widely studied and fairly well understood. However, this approximation is not sufficient to model the turbulent flow in the trailing edge during flight. Unsteadiness is an inherent part of the flow during flight, where there are far more variables that can change and cannot be accounted for during wind tunnel steady studies made on airfoils, hence, however the static stall and aerodynamics coefficients prediction are made considering well-developed flow past the airfoil in a quasi-steady condition it is necessary to do an unsteady analysis in order to capture dynamic stall characteristics.

Most aeronautical devices can encounter unsteady flow behavior during flight, whether designed for that or not. The unsteadiness is easily observable in the flow of rotorcraft devices and wind turbines, but also it is present in fixed wings, which can vibrate at high frequencies during flight, leading to undesirable problems as the flutter phenomena. The unsteadiness is also a way to delay dynamic stall to control periodic vortex generation and improve the performance of rotorcrafts and wind turbines (McCroskey, 1982).

Unsteady effects are also evident in the most fascinating spectacle, the flight of birds, which are not yet fully understood due to the challenges in creating and simulating a device that can reproduce all the motion characteristics of bird's flapping wings. Nowadays, the modelling of flapping wings, or flapping airfoils when considering a two dimensional approach, is the combination of a plunging (or heaving) and pitching (oscillatory) motion. Considering this kind of motion, it is possible to see beneficial effects of unsteadiness, which is substantially important to the propulsive efficiency of flapping motion.

As a component of a flapping motion, a pitching airfoil is a simple way to study and can be used to assess the influence of the trailing edge vortex on flow reattachment. Due to the flow fluctuation, the circulation near the wall varies, which is accompanied by the shedding of free

vorticity into the wake. When the pitching airfoil reaches a high incidence angle, the vortex has a high energy profile, causing vortex shedding at the leading edge and reaching dynamic stall, causing large loss in the lift and increasing drag. As the incidence angle decreases downstroke, the flow reattaches, delayed as compared to static stall, as found in the experiments of McAlister *et al.* (1978) and Berton *et al.* (2002).

These unsteady effects can also be related to aeroelastic effects, where mutual interaction between aerodynamics and elastic forces on lifting surfaces is investigated. The aeroelastic effects are observable since the first attempts of flight, being the cause of many unsuccessful flights in the beginning of the 20th Century. The first model to linearize the aeroelastic effects was based on small disturbance theory, by Theodore Theodorsen (1934), where the model coupled unsteady aerodynamics effects modelled by using the two-dimensional Bernoulli equations in a typical section, resulting in the first aeroelastic investigations (Johansen, 1999). Theodorsen then formulated a solution for the dynamic pitching airfoil in order to understand the flutter phenomenon.

The static stall can be characterized with the formation of a leading edge vortex and a laminar separation bubble close to the leading edge region, where the vortex travels along the surface and starts to grow. The static stall finally occurs when the vortex separates from the airfoil as it reaches the region close to the trailing edge.

Differently, in the dynamic stall the airfoil flow separation occurs at a higher angle-of-attack, where it can be characterized as a sudden drop of lift during the dynamic pitching motion. The shear layer near the leading edge rolls up to form the leading-edge vortex providing additional suction over the upper surface of the airfoil as it moves in the trailing edge direction. This additional suction allows a delay in the separation of the boundary layer. However, the boundary layer becomes unstable as the airfoil reaches higher incidence angles, leading to the dynamic stall. Dynamic stall is not a well-understood phenomenon yet, even when considering its importance to the performance and operational limits of helicopters, flapping wings, and wind turbines (McCroskey, 1982).

2 Background Theory

This chapter will present the literature review on the study and analysis of pitching airfoil flows as well as the underlying theory on dynamic stall and unsteady aerodynamics. It is important to firstly understand the mechanism of dynamic stall and the stages that can characterize it. Also, the main studies on the area are presented as a short description of the most important insights found by the researchers.

2.1 The Dynamic Stall

The static stall of an airfoil is an aerodynamic phenomenon characterized by flow separation and the decrease of lift. For steady airfoils, the flow encounters its wall and attaches to it, forming a laminar boundary layer. In the transitioning from laminar to turbulent flow, a separation bubble is formed close to the airfoil leading edge, creating a reverse flow region. Afterwards the boundary layer is reattached into turbulent flow until it reaches the trailing edge turbulent separation point, and thus creates a separated turbulent shear layer and the detached turbulent separation region. As the incident angles of the airfoil increases, the trailing edge separation point progress upward in the upper surface until it reaches the transition bubble. At this point the flow does not reattach after the laminar separation, the bubble “bursts” and then a separated turbulent flow region is created, leading to the stall of the airfoil by losing lift. The flow behaves in a different way for unsteady airfoils, since the vertical wake is now time-dependent due to the unsteadiness and the aerodynamics coefficients can change accordingly.

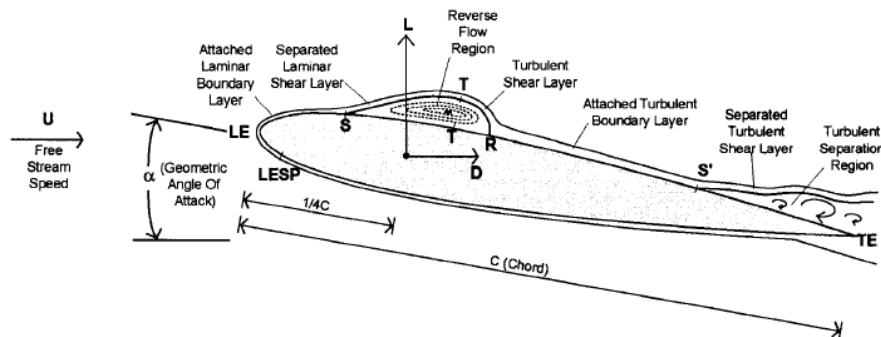


Figure 2.1-1 Flow structure analysis for static NACA 0012 airfoil (Gerontakos, 2004)

The stall of a body under unsteady motion is quite complex when compared to static stall (McCroskey, 1982). Dynamic stall is a phenomenon that occurs on lifting surfaces when the angle of attack increases at a finite rate until stall is produced, differently from the static stall since the angular velocity leads to the pitching motion and allows the airfoil to reach higher incidence angles than the one observed on the static case. The dynamic stall of a pitching motion can be characterized as either a rapid increase into the stall region or a harmonic oscillatory motion that leads to periodic vortex generation and stall and unstalling of the airfoil (Malone, 1974). As the pitching airfoil surpasses its static stall angle the flow is still attached to its surface and in a higher incidence angle it is possible to observe the appearance of reversal flow on its surface close to the trailing edge. As it moves upstroke ($\uparrow$), large eddies appear in the boundary layer and the reversal flow starts to spread over the upper surface and a large leading edge vortex will appear causing the surge of the lift slope (Fig. 2.1-2).

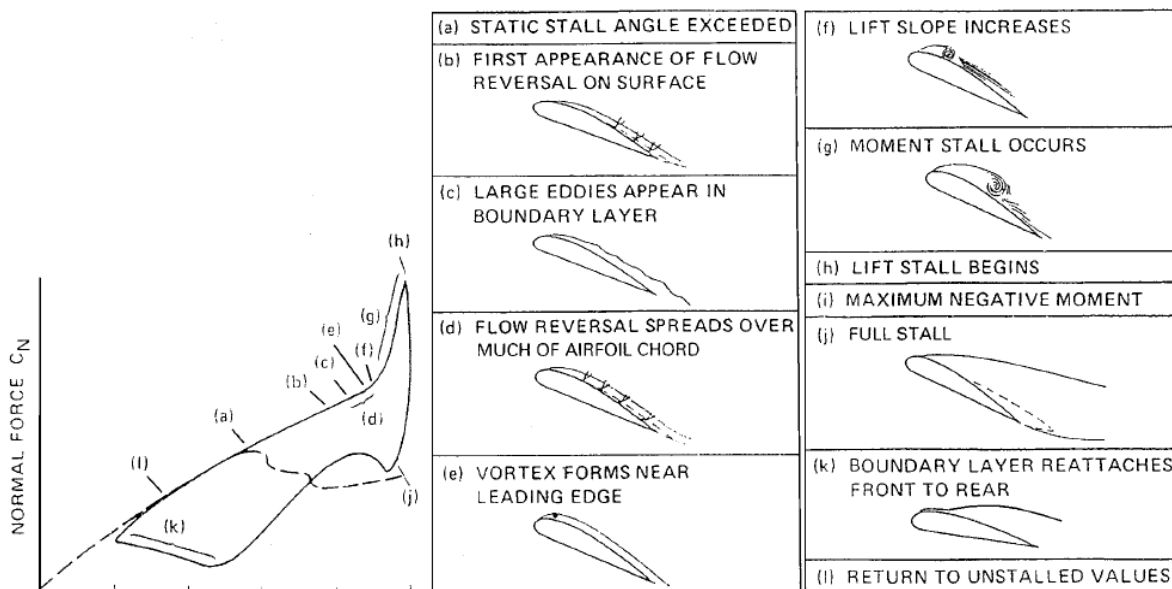


Figure 2.1-2 Dynamic stall stages for a pitching motion NACA0012 airfoil, adapted from Carr (1988) - Analysis of normal force and pitching moment coefficients

The flow will then reach its dynamic stall condition and during the downstroke phase ($\downarrow$) the airfoil will be full stalled. At low incidences during the downstroke the flow begins to stabilize and the boundary layer reattachment process starts. Depending on the Reynolds number the reattachment process can be fast enough to end before the airfoil reaches its minimum

incidence due to the flow acceleration (high Reynolds) or can be slow, finishing later on the upstroke phase of the pitch cycle to follow.

2.2 Leading Studies on Dynamic Stall and Pitching Airfoils

Numerous studies were made regarding the unsteady aerodynamics since the beginning of the 1930's, as the first theories regarding the linearization of an unsteady function started. Due to the advance of measurement systems, experimental analysis was made in the late 1970's and the 2000's to investigate the periodic vortex formation and compare the flow behavior with respect to static results. Also, during the end of the 20th and beginning of the 21st Century, the advance on the computational capabilities and the developing of numerical methods turbulence models allowed the solution of the Navier-Stokes equations via Computational Fluid Dynamics (CFD), helping on the analysis of unsteady flows. These studies focused not only on the understanding of vortex formation but also the solution for problems related to the aeronautical industry, such as the flutter phenomena, as well as a way to improve turbomachinery, rotorcrafts and wind turbines performance as the flow unsteadiness is a way to control stall and the periodic vortex formation. A quite interesting approach would be the use of pitching airfoils as a future thrust generation device, as idealized by McCroskey (1982) and Muller *et al.* (2008).

The dynamic pitching motion of an airfoil is related to aeroelastic problems, which led to the development of steady and unsteady aerodynamics. Theodorsen (1934) formulated his General Theory of Aerodynamic Instability and The Mechanism of Flutter to study the phenomena, formulating a solution for the dynamic pitching airfoil motion. During the pitching motion the wake behind the airfoil affects the velocity field close to the wall and the forces acting on it, also the wake can be characterized as an inviscid and a viscous part. In the latter the viscous wake is limited to a very thin region at low Reynolds Numbers, $\sim 10^5$ (Yang *et al.*, 2006). The angle of attack dynamic change of the airfoil during the pitching motion is responsible to the inviscid wake, leading to the vortical structures encountered in the flow.

To understand the flow characteristics during pitching motion many experimental studies were made in the end on the 1970's in an attempt to model the flow behavior and encounter a linearized oscillating-airfoil theory. One of the main studies on this area was made by McAlister, Carr & McCroskey (1978), who performed an extensive study on dynamic stall for pitching

airfoils, finding that the delay of the boundary layer separation is due to the residence of the shed vortex on the airfoil upper surface, due to the unsteady behavior of the flow. The experiments were performed on a NACA 0012 airfoil oscillating in pitch at Reynolds Number 2.5×10^6 and Mach number 0.09. The data acquisition system used hot-wire probes and surface-pressure transducers to clarify the role of laminar separation bubble to delineate the growth of stall vortex shedding and quantify the aerodynamic loads.

It was found that the laminar separation bubble at the leading edge has no effect on the overall dynamic stall, where the vortex shedding at the trailing edge is predominant on the flow characteristics. Also, the strong lift surge is an induced effect from the shed vortex during its period of residence over the airfoil. The studies from McAlister *et al.* (1978) were also important on the characterization of the stall depending on its kind, which can be two: the fully developed stall and the partially developed stall. The first occurs when the vortex is shed while the airfoil is still pitching up (upstroke phase), leading to a more abrupt stall. The second describes the dynamic stall at maximum incidence, where the phenomenon is smoother.

McCroskey (1982) not only studies the main effects on pitching airfoils but also presents some modifications to the Theodorsen's theory. The work states that many of unsteady airfoil behavior can be described by linearized thin-airfoil theory. The fluid-dynamic pressure forces acting on a thin lifting surface inclined at a small angle relative to the approaching flow are proportional to the effective angle of attack and to the square of the speed of the flow. If either the body or the flow fluctuates, so do the circulation and the pressure distribution; and each change in circulation around span wise sections of the body is accompanied by the shedding of free vorticity from the trailing-edge region into the wake. This time-dependent vortical wake is an important distinguishing feature of unsteady airfoils. During the pitching motion the airfoil can reach high angles of attack, past the static-stall limit, until it reaches the dynamic stall incidence, characterized by a massive separation and generation of large vortical structures, responsible for the unsteady separation and reattachment. At higher incidence angles the vortex presents a high energy profile, causing vortex shedding in the leading edge until the airfoil reaches its dynamic stall. Due to the flow fluctuation, the circulation near the wall varies, which is accompanied by the shedding of free vorticity into the wake. The behavior is also periodic, after stabilizing the flow will present almost same characteristics for each cycle, which is due to the harmonic motion of the airfoil.

On the work from Srinivasan, Ekaterinaris & McCroskey (1995) the numerical analysis for the dynamic stall was studied for five turbulence models: the Baldwin-Lomax algebraic model; the Renormalization Group Theory (RNG) based algebraic model; the half-equation Johnson-King model; and the one-equation models of Baldwin-Barth and Spalart-Allmaras for the analysis of the two-dimensional flowfield of an oscillating NACA 0015 airfoil. It is shown that the RNG, Johnson-King and Spalart-Allmaras models have good agreement with experimental results for the aerodynamics coefficients. For all the models there was an overprediction of the extent of separation, though the upstroke was well represented. There was a good qualitative agreement for the downstroke phase. Ekaterinaris & Platzer (1997) continued the work from Srinivasan *et al.* (1995) in a more extensive analysis evaluating the computational capabilities on modelling unsteady pitching flows for both two-dimensional and three-dimensional simulations, including now two-equation turbulence models as the $k-\epsilon$ and $k-\omega$. The advance on turbulence modelling led to a significant improvement on the numerical computational of dynamic stall; however it was seen that some problems on the modelling of flow reattachment process and the incorporation of transitional effects.

Advancements made on data acquisition devices in the end of the 1990's, as PIV (Particle Image Velocimetry) and LDV/LDA (Laser Doppler Velocimeter/Anemometer), were essential to improve experimental measurements and wake studies, as shown by Berton *et al.* (1997, 2002) and Maresca *et al.* (2000). The experimental analysis by Berton *et al.* (2002) focused on the use of Laser Sheet Visualization Method (LSV) for the study of the boundary layer behavior and periodic separation and reattachment process for an oscillating NACA 0012 airfoil, Reynolds number 10^5 and 2×10^5 . Also the study used Embedded LDV (ELDV) as a tool for capturing instantaneous velocity components on the upper surface of the airfoil. The analysis shows that the LSV is a suitable and useful tool on the investigation of unsteady boundary layer on oscillating airfoils, being able to show the different flow features occurring during downstroke and upstroke, as the visualization of the large separation bubble and high vortical flow.

Also, the advancements on computing memory operations and development of URANS methods, OES and LES methods, were essential to the turbulence modelling of unsteady airfoils. The use of classical URANS models, as the $k-\epsilon$ Chien and the Spalart-Allmaras turbulence model, and the hybrid URANS-LES turbulence modelling by using a Delayed Detached Eddy Simulation (DDES) is shown by Martinat *et al.* (2008). Kasibhotla & Tafti (2014) presented the

use of Wall Resolved LES to model the flow over an oscillating airfoil. The analysis made on both studies shows that even with the advance of the numerical methods there are still some problems to overcome since as presented by Ekaterinaris & Platzer (1997) there is a clear overestimation of lift by some extent during the upstroke. Also, the dynamic stall incidence is not accurately modelled, as for the boundary layer reattachment process during downstroke phase.

2.3 The Theodorsen's Function

As commented, the main reasoning for the study of unsteady aerodynamics was aeroelastic related problems, such as the flutter phenomena. The first theoretical solutions started being formulated in the beginning of the 1920's and culminated on the *General Theory of Aerodynamic Instability and The Mechanism of Flutter* from Theodorsen (1934), and many other studies and attempt of linearization of dynamic stall models, as the work from Von Kármán & Sears (1938). The airfoil was then considered as a thin plate and a trailing flat wake of vorticity of incompressible fluid, following the conditions from Neumann, Kutta and Kelvin. When considering periodic oscillations the flow can be characterized by the non-dimensional frequency parameter, the reduced frequency, as presented in Eq. (2.1).

$$k = \omega c / 2U_{\infty} \quad (2.1)$$

The solution of the Theodorsen's Functions can be expressed in terms of combinations of standard Bessel functions whose argument is k . In the reduced frequency equation, ω is the circular frequency of oscillation, c is the chord of the airfoil and U_{∞} is the mean free-stream velocity. As explained in the work from McCroskey (1982), for a sinusoidal oscillation in pitch, it is possible to consider a flat-plate airfoil defined by $0 \leq X \leq 1$ and it oscillates harmonically in pitch about an axis located at $X = A$. The angle of attack is defined as $\alpha = \alpha_o \operatorname{Re} \cdot e^{i\omega t}$.

In his theory, Theodorsen divided the flow into two components: the noncirculatory component of sources and sinks, satisfying the boundary conditions on the oscillating plate and that includes the apparent mass effects; and the circulatory component, which includes bound

vortices and wake vortices. The circulatory and noncirculatory components are matched at the trailing edge of the airfoil to enforce the Kutta Condition of nonsingular flow. Therefore, the aerodynamics coefficients of lift, pressure and moment about the pitch axis can be expressed:

$$C_p = -2\alpha \sqrt{\frac{1-X}{X}} [f_1 - ig_1] \quad (2.2)$$

$$C_L = 2\pi\alpha [f_2 - ig_2] \quad (2.3)$$

$$C_M = 2\pi\alpha \left(A - \frac{1}{4}\right) [f_3 - ig_3] \quad (2.4)$$

The unsteady effects are included in the functions $f_n + ig_n$, where Theodorsen apply the Bessel functions of k , and A to model the unsteadiness. A simple approach of the Theodorsen's function will be presented following the theory presented in the lecture notes from the Aircraft Loads and Aeroelasticity course (FEMEC43080, UFU) by Guimarães (2015).

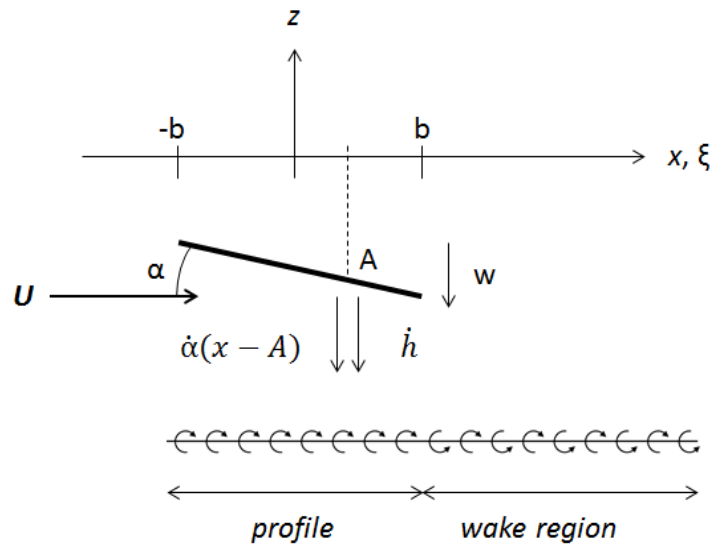


Figure 2.3-1 Vortex model for a pitching flat-plate

The pitching airfoil can be represented as a pitching flat-plate with a vortex distribution

projected on a line parallel to the flow, as shown in Fig. 2.3-1. The motion of the airfoil is then represented by a plunging component, $\dot{h}$ and a pitching component $\dot{\alpha}$. The unsteady Bernoulli equation can be given as it follows.

$$\Delta p(x, t) = \rho \left[U_{\infty}(t) \gamma(x, t) + \frac{\partial}{\partial t} \int_0^x \gamma(\xi, t) d\xi \right] \quad (2.5)$$

Where $\gamma(x, t)$ is assumed as the vortex sheet strength. The unsteady lift force on the airfoil can be calculated as Eq. (2.6), where is possible to expand mathematically the Eq. (2.5).

$$L = \int_0^c \Delta p(x, t) dx = \rho U_{\infty}(t) \int_0^c \gamma(x, t) dx + \rho \frac{\partial}{\partial t} \int_0^c (x - c) \gamma(x, t) dx \quad (2.6)$$

The first term of the equation corresponds to the Kutta-Joukowski Theorem, being the integral the equation for the circulation Γ over the airfoil. The second part is the apparent mass effect, from the noncirculatory component of the flow. From the Biot-Savart Law, the downwash at $z=0$ can be described as it follows, being γ_a the vortex sheet strength at the profile and γ_w the vortex sheet strength in the wake.

$$w(x, 0, t) = -\frac{1}{2\pi} \int_{-b}^{+b} \frac{\gamma_a(\xi, t)}{x - \xi} d\xi - \frac{1}{2\pi} \int_{-b}^{+b} \frac{\gamma_w(\xi, t)}{x - \xi} d\xi \quad (2.7)$$

And from the analysis of the velocity components from Fig. 2.3-1, the downwash at the trailing edge can be written as:

$$w = U_{\infty}(t) \alpha(t) + \dot{h}(t) + \dot{\alpha}(t)(x - A) \quad (2.8)$$

The downwash in the airfoil can also be related to the components of the vortex sheet at the flat-plate and the wake vortex, as in Eq. (2.9).

$$w(x, t) = w_a(x, t) + w_w(x, t) \quad (2.9)$$

The flat-plate solution needs to satisfy the following conditions:

- The flow over the airfoil has to be tangent to its surface, hence the flow will not cross the airfoil wall, ensuring the Neumann Condition;
- The vorticity must be zero at the airfoil trailing edge, ensuring Kutta Condition, as the circulation Γ is such that the flow leaves the trailing edge smoothly;
- The total nonstationary circulation Γ is conserved, following Kelvin's Circulation Theorem, where $D\Gamma/Dt = 0$.

From the Kelvin's Circulation Theorem is then possible to state that:

$$\Gamma(t) = \Gamma_a(t) + \Gamma_w(t) = cte \quad (2.10)$$

From Neumann Condition, the vortex strength can now be represented considering:

$$w(x, t) - w_a(x, t) - w_w(x, t) = 0 \quad (2.11)$$

$$U_\infty(t)\alpha(t) + \dot{h}(t) + \dot{\alpha}(t)(x - A) - w_w(x, t) = -\frac{1}{2\pi} \int_{-b}^{+b} \frac{\gamma_a(\xi, t)}{x - \xi} d\xi \quad (2.12)$$

Applying the Kutta Condition at the trailing edge and considering mathematic operations it is possible to determine:

$$\gamma_a(+b, t) = 0 \quad (2.13)$$

$$\gamma_a(x, t) = -\frac{2}{\pi} \sqrt{\frac{b-x}{b+x}} \int_{-b}^{+b} \sqrt{\frac{b-\xi}{b+\xi}} \cdot \frac{w_a(x, t) - w_w(x, t)}{x - \xi} d\xi \quad (2.14)$$

Theodorsen verified that the vorticity associated to flow circulation at the airfoil can be described as two components:

- The Circulatory Vorticity γ_a^C , which is the contributing component to the circulation Γ and has no effect over the downwash at the airfoil trailing edge;
- The Noncirculatory Vorticity γ_a^{NC} , under the conditions previously stated, however does not contribute to Γ .

Hence, the lift can be found relating Eq. (2.6) and Eq. (2.14), being now divided in the components of the quasi stationary parcel, the induced lift parcel and the noncirculatory lift, as described in Eq. (2.15).

$$L = (L_{QS} + L_W) + L_{NC} = L_C + L_{NC} \quad (2.15)$$

The terms of circulatory and noncirculatory lift are given as it follows:

$$L_{QS} = 2\rho\pi b U_\infty(t) \left[U_\infty(t)\alpha(t) + \dot{h}(t) + \dot{\alpha}(t) \left(\frac{3b}{2} - A \right) \right] \quad (2.16)$$

$$L_W = \rho U_\infty(t) \int_b^\infty \frac{b}{\sqrt{\xi^2 - b^2}} \gamma_w(\xi, t) d\xi \quad (2.17)$$

$$L_{NC} = \rho\pi b^2 [U_\infty(t)\dot{\alpha}(t) + \ddot{h}(t) + \ddot{\alpha}(t)(b - A)] \quad (2.18)$$

From Eq. (2.15) to (2.18) the lift over the airfoil can finally be written as described in Eq. (2.19).

$$L(t) = \frac{\int_b^\infty \frac{\xi}{\sqrt{\xi^2 - b^2}} \gamma_w(\xi, t) d\xi}{\int_b^\infty \sqrt{\frac{\xi + b}{\xi - b}} \gamma_w(\xi, t) d\xi} \cdot L_{QS} + L_{NC} \quad (2.19)$$

Considering the harmonically plunging and pitching of the flat-plate, for the wake it is possible to consider:

$$\bar{\xi} = \frac{\xi}{b} \quad (2.20)$$

$$\gamma_w(\xi, t) = \bar{\gamma}_w e^{i(\omega - k\bar{\xi})} \quad (2.21)$$

By substituting Eq. (2.21) in Eq. (2.19) is possible then to find the Theodorsen's Function as a function of the reduced frequency of the flow.

$$C(k) = \frac{\int_1^\infty \frac{\bar{\xi}}{\sqrt{\bar{\xi}^2 - 1}} e^{-ik\bar{\xi}} d\bar{\xi}}{\int_1^\infty \sqrt{\bar{\xi} + 1 / \bar{\xi} - 1} e^{-ik\bar{\xi}} d\bar{\xi}} \quad (2.22)$$

Theodorsen identified the integrals as a combination of Hankel functions of second kind:

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \quad (2.23)$$

Finally, the lift expression can be rearranged to a simple function of the Theodorsen's function and the components of plunging and pitching of the airfoil, as described in the following equation.

$$\begin{aligned} L(t) = & \rho\pi b^2 [U_\infty(t)\dot{\alpha}(t) + \ddot{h}(t) + \ddot{\alpha}(t)(b - A)] \\ & + 2\rho\pi b U_\infty(t) C(k) \left[U_\infty(t)\alpha(t) + \dot{h}(t) + \dot{\alpha}(t) \left(\frac{3b}{2} - A \right) \right] \end{aligned} \quad (2.24)$$

Considering the pitching motion only, the equation of the lift can be written as:

$$L(t) = \rho\pi b^2[U_\infty(t)\dot{\alpha}(t) + \ddot{\alpha}(t)(b - A)] + 2\rho\pi b U_\infty(t)C(k) \left[U_\infty(t)\alpha(t) + \dot{\alpha}(t)\left(\frac{3b}{2} - A\right) \right] \quad (2.25)$$

The pitching lift coefficient can then be written as:

$$C_l(t) = \frac{\pi b}{U_\infty^2(t)} [U_\infty(t)\dot{\alpha}(t) + \ddot{\alpha}(t)(b - A)] + \frac{2\pi C(k)}{U_\infty(t)} \left[U_\infty(t)\alpha(t) + \dot{\alpha}(t)\left(\frac{3b}{2} - A\right) \right] \quad (2.26)$$

One can observe from Eq. (2.26) that the lift coefficient, as well for drag and pitching moment coefficients (not described in the analysis), depends on several parameters as the reduced frequency; free-stream velocity and the pitch derivatives, flow parameters as well as airfoil chord and the pitch axis position, geometric parameters. However, as the theory approaches the unsteady motion as a flat-plate pitching and plunging, the influence of the airfoil camber is not accounted. Thus, a different approach is necessary to model the unsteady motion of an airfoil. The influence of the reduced frequency and the pitch axis position on the pitching behavior is present on Fig. 2.3-2.

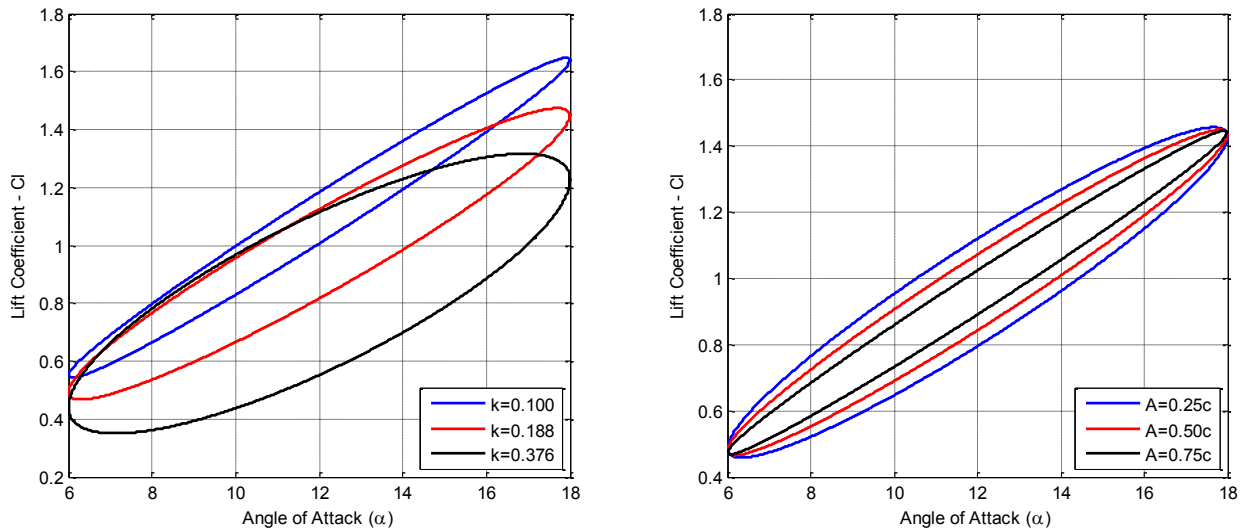


Figure 2.3-2 Lift Coefficient curves from the Theodorsen's function for a flat-plate in pure pitch motion. The hysteresis loop enlarges and moves down with the increase of the reduced frequency (left) and thins as the pitch axis moves far from the leading edge (right)

3 Goals

Although the Theodorsen's function can explain the hysteretic behavior for the airfoil lift during an unsteady harmonic motion it cannot accounts for the rotational behavior of the fluid that can be encountered during unsteady motion. Also, the Theodorsen's function is limited to low Reynolds numbers, differently from what aircraft and wind turbines can encounter during operation. Experimental analysis from ELDV measurements, as the study from Berton *et al.* (2002), have shown that the downstroke phase for the harmonic pitching motion presents a tridimensional behavior, which is not accounted by the Theodorsen's function as well. Hence, new methods are necessary to model flows closer to operational conditions, characterized by higher order Reynolds numbers and rotational behavior, as the flow is expected to be turbulent.

The purpose of this study is to model the pitching motion of a NACA 0012 airfoil and evaluate the capabilities of URANS $k-\omega$ SST model on modelling unsteady pitching flows, which is an approximation of the vibration that can be faced by aerodynamic bodies during flight, for different Reynolds numbers. The two-dimensional CFD computations here presented are based on the experimental studies from Berton *et al.* (2002) and McAlister *et al.* (1978), for Reynolds number 10^5 and 2.5×10^6 , respectively.

The Unsteady Reynolds-averaged Navier–Stokes equations (URANS) are solved by the finite volume solver Fluent 14.5 (ANSYS, Inc., 2012). The Shear Stress Transport SST model (Menter, 1993, 2003) was used for the numerical simulations, since it can accurately predict the flow at both sublayer and boundary layer edge and works well at capturing recirculation regions and to accurately predict the adverse pressure gradient related to unsteady effects of transient flows. Since the SST is a blend of the $k-\omega$ model, for near wall sublayer prediction, and the $k-\epsilon$, for freestream prediction, it is necessary to transform the former into equations based on k and ω .

4 Mathematical Modeling

From the literature review it is possible to understand that this kind of problem can be approached using many different schemes, from one and two equation models to higher order methods. However the complexity of this kind of flow requires an approach that is able to capture its characteristics, which can be difficult to model. Hence, once must choose a simulation technique that should model the problem with the necessary accuracy in an efficient way.

This chapter presents the mathematical modeling employed on the current study. The flow is modeled using the URANS method with the k- ω SST turbulence model, as will be detailed in the following sections.

4.1 Governing Equations

The incompressible Newtonian fluid flow can be described by the use of differential equations, where the flow is modeled by the Continuity and the Navier-Stokes Equations. These equations can be described as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (4.1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left[(\mu + \mu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + f_i \quad (4.2)$$

The Continuity equation (Eq. 4.1) states that mass must be conserved through the boundaries of a control volume. The Navier-Stokes equation for momentum is described in Eq. (4.2).

4.2 The Unsteady Reynolds-averaged Navier-Stokes

To model industrial flows is often recommended the use of a fast yet accurate method. An accurate LES simulation would require a very fine mesh to avoid problems near walls, which can

demand time and great computational effort. One approach that can make good use of a less refined mesh and without consuming much computational effort is the unsteady RANS, often denoted URANS or Transient RANS, TRANS (Davidson, 2006, MTF270 Chalmers).

The technique uses a statistical, averaged, approach to approximate the Navier-Stokes equation, as proposed by Reynolds (1895). Time averaging is appropriate for stationary turbulence, as described by Wilcox (2006), which is a turbulent flow that, on the average, does not vary with time. The pitching flow is by definition unsteady; however, when considering a harmonic behavior of the pitch motion, it presents periodic characteristics that only can be modeled by an unsteady method. Hence the Unsteady RANS approach is suitable for the analysis.

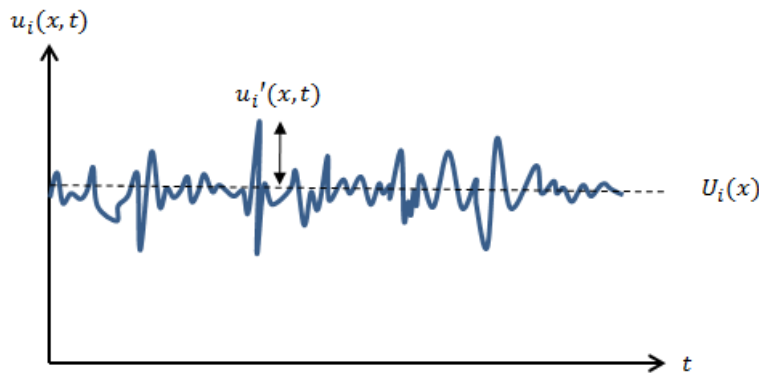


Figure 4.2-1 Time averaging for stationary turbulence

As described by Reynolds, the instantaneous velocity field $u_i(x, t)$ of the flow can be expressed as the sum of a mean velocity, $U_i(x)$, and a fluctuating part, $u_i'(x, t)$. This is known as Reynolds Decomposition.

$$u_i(x, t) = U_i(x) + u_i'(x, t) \quad (4.3)$$

The mean velocity is denoted by:

$$U_i(x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} u_i(x, t) dt \quad (4.4)$$

As one can expect, the time-average of the mean velocity is again the same mean velocity:

$$\bar{U}_i(x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} U_i(x) dt = U_i(x) \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} dt = U_i(x) \quad (4.5)$$

This leads to:

$$\overline{u'_i} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} [u_i(x, t) - U_i(x)] dt = U_i(x) - \bar{U}_i(x) = 0 \quad (4.6)$$

Flows for which the mean flow contains very slow variations with time, as when it is necessary to compute the flow over a helicopter blade, for example, which can be described as a nonstationary turbulence flow.

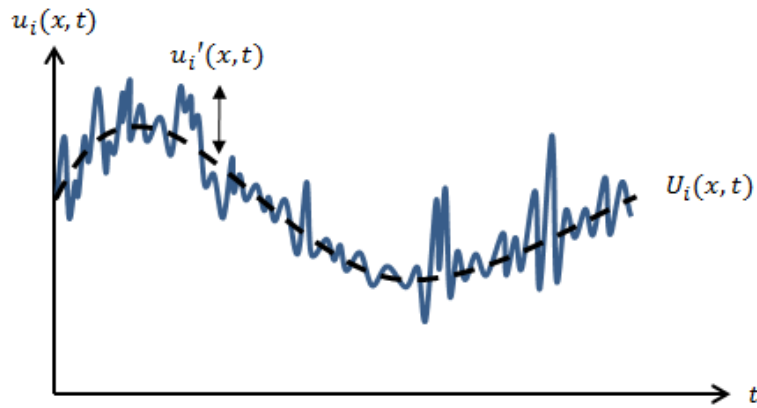


Figure 4.2-2 Time averaging for nonstationary turbulence

To achieve a time varying approach, one must consider:

$$u_i(x, t) = U_i(x, t) + u'_i(x, t) \quad (4.7)$$

and,

$$U_i(x, t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} u_i(x, t) dt \quad (4.8)$$

Which is the simplest approach, yet arbitrary. As stated before, the time-averaging of the fluctuations is still zero, thus, for any scalar p and vector u_i it is possible to write:

$$\overline{p_{,i}} = P_{,i} \quad \& \quad \overline{u_{i,j}} = U_{i,j} \quad (4.9)$$

Yielding:

$$\frac{\partial \overline{u_{i,j}}}{\partial t} = \frac{\partial U_{i,j}}{\partial t} \quad (4.10)$$

As stated by Wilcox (2006), the use of time-averaging is useful for the analysis, especially for steady flows. A degree of caution must be accounted when considering time-varying flows, though. This is mainly due to the fluctuations that are often in excess of 10% of the mean velocity of the flow. For the analysis of the Reynolds-averaged equations, Eq. (4.1) and Eq. (4.2), for incompressible flow, must be rewritten as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (4.11)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial (u_i u_j)}{\partial x_j} = - \frac{\partial p}{\partial x_j} + \frac{\partial t_{ji}}{\partial x_j} \quad (4.12)$$

Where t_{ij} is the viscous stress tensor as it follows:

$$t_{ij} = 2\mu s_{ij} \quad (4.13)$$

Being μ the molecular viscosity and s_{ij} the strain-rate tensor, given as:

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (4.14)$$

Combining Eq. (4.12) through Eq. (4.14) brings the conservation form of the Navier-Stokes equations:

$$\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial (u_i u_j)}{\partial x_j} = - \frac{\partial p}{\partial x_j} + \frac{\partial (2\mu s_{ji})}{\partial x_j} \quad (4.15)$$

Applying the time-averaging to Eq. (4.11) and Eq. (4.15), considering also Eq. (4.9):

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (4.16)$$

$$\rho \frac{\partial U_i}{\partial t} + \rho \frac{\partial (U_i U_j - \overline{u'_i u'_j})}{\partial x_j} = - \frac{\partial P}{\partial x_j} + \frac{\partial (2\mu \overline{s_{ji}})}{\partial x_j} \quad (4.17)$$

Where by rearranging Eq. (4.17) it is possible to write the momentum equation in tensor notation, as it follows:

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = - \frac{1}{\rho} \frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\nu \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) - \overline{u'_i u'_j} \right] \quad (4.18)$$

The Reynolds-averaged Navier-Stokes equation is then described by Eq. (4.18). The term $-\overline{u'_i u'_j}$ is also known as the Reynolds-stress tensor, τ_{ij} , which has to be modeled in order to solve the RANS equations. This modeling is known as turbulence modeling. For general three-dimensional flows the described equations give four unknown mean-flow properties, namely the pressure and the three velocity components. Along with the six Reynolds-stress components it yields ten unknowns for four equations. Hence the system is not closed and it is necessary to find enough equations to solve the system, as necessary in turbulence modeling.

4.3 The Closure Problem

To model turbulence it is necessary to find an approximation for the Reynolds-stress tensor. Boussinesq (1877) proposed a solution by assuming that the Reynolds-stresses are proportional to the velocity gradients of the mean-flow, analogous to the viscous stresses, yielding:

$$\tau_{ij} = -\overline{u'_i u'_j} = -\nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij} \quad (4.19)$$

The kinetic energy k and the kinematic Eddy viscosity ν_t are unknown, since:

$$k = \frac{1}{2} \overline{u'_i u'_i} = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \quad (4.20)$$

And the kinematic Eddy viscosity is modeled by introducing extra equations. For RANS simulations two of the most used turbulence models are the k- ϵ and k- ω , where both solve two extra transport equations, one for k and one for ϵ or ω . The kinematic viscosity is modeled by the k- ϵ as:

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \quad (4.21)$$

and the k- ω as:

$$\nu_t = \frac{k}{\omega} \quad (4.22)$$

This study uses the k- ω SST turbulence model in order to solve the NSE. This model is a blend of the two models described previously, applying the k- ω model in the near-wall region and the k- ϵ to model the free-stream flow.

4.4 Turbulence Modeling

Aeronautical flows are surely a class where the prediction of its properties needs high accuracy, mainly due to strong adverse pressure gradients and separation in boundary layers. The k - ϵ and k - ω two-equation RANS models are not able to capture the proper behavior of turbulence in aeronautical flow. The popular k - ϵ can give a well-defined boundary layer-edge during simulation; however, it is less accurate and complex on sublayer modelling. The k - ω is substantially more accurate in the sublayer; yet it is sensitive in the freestream, which is the cause of the k - ϵ being the standard equation in turbulence modelling (Menter *et al.*, 2013). Both standard two-equation models overpredict the shear stress in adverse pressure gradient flows, even when considering delayed separation. The Shear Stress Transport SST model (Menter, 1993) was developed due to the need of more accurate separation prediction for aeronautic flows. The k - ω SST model is a blend of a k - ω model, which is used near walls in the sublayer prediction, and a k - ϵ model, used to predict the flow in the freestream region. Thus, the model is fairly robust, since it can accurately predict the flow at both sublayer and boundary layer edge and works better at capturing recirculation regions by enforcing the Bradshaw Relation.

To blend the k - ϵ and k - ω models, it is necessary to transform the former into equations based on k and ω . This leads to the cross-diffusion term, defined in Eq. (4.23).

$$D_w = (1 - F_1)\rho \frac{2}{\sigma\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \quad (4.23)$$

The blending function for the model is defined, assuming value zero in the freestream region, activating the k - ϵ cross diffusion term, and switches to one in the boundary-layer zone, to assure accurate calculation by the use of the k - ω function.

$$F_1 = \tanh \left\{ \left\{ \min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{4\rho \sigma_{\omega^2} k}{C D_{k\omega} y^2} \right] \right\} \right\} \quad (4.24)$$

Where y is the distance to the nearest wall and:

$$CD_{k\omega} = \max\left(2\rho\sigma_{\omega^2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 10^{-10}\right) \quad (4.25)$$

The SST Turbulence Kinect Energy function is given as shown in Eq. (4.26). The Dissipation Rate, combining both standard k- ϵ and k- ω models by the use of the cross-diffusion term and the blending function F1 is shown at Eq. (4.27), being both the equations of this class of RANS model.

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_i k)}{\partial x_i} = \tilde{P}_k - \beta^* \rho k \omega + \frac{\partial}{\partial x_i} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_i} \right] \quad (4.26)$$

$$\begin{aligned} \frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho U_i \omega)}{\partial x_i} \\ = \alpha \rho S^2 - \beta^* \rho \omega^2 + \frac{\partial}{\partial x_i} \left[(\mu + \sigma_{\omega} \mu_t) \frac{\partial \omega}{\partial x_i} \right] + 2(1 - F_1) \rho \sigma_{\omega^2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \end{aligned} \quad (4.27)$$

The turbulent Eddy viscosity is defined as:

$$\nu_t = \frac{a_1 k}{\max[a_1 \omega, SF_2]} \quad (4.28)$$

Where S is the strain rate and F2 is a second blending function defined by Eq. (4.29). The model uses a production limiter to avoid turbulence build-up in stagnation regions, as Eq. (4.30).

$$F_2 = \tanh \left\{ \left[\max \left(\frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right) \right]^{-2} \right\} \quad (4.29)$$

$$P_k = \mu_t \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \rightarrow \tilde{P}_k = \min(P_k, 10\beta^* \rho k \omega) \quad (4.30)$$

During the upstroke of a pitching motion, the effect of an adverse pressure gradient is limited, and then it leads to a dynamic stall angle that exceeds the experimental static stall, which

increases the aerodynamic coefficients. However, during the downstroke, the adverse pressure gradient largely increases, leading to a reattachment angle that is smaller than that of the static case. As shown above, the $k-\omega$ SST was designed to capture recirculation and to accurately predict the adverse pressure gradient related to unsteady effects of transient flows. Nevertheless, since the analysis is two-dimensional, it is possible that the prediction will not be fully correct.

5 Numerical Setup

As stated in Chapter 3, the airfoil geometry for the numerical simulations is the NACA 0012, in accordance with the experiments from Martinat *et al.* (2002) and McAlister *et al.* (1978), for chord length based Reynolds number 10^5 and 2.5×10^6 , respectively. The airfoil will pitch at the quarter chord position aft the leading edge, also known as both center of pressure and aerodynamic center for symmetric airfoils, as presented in the *Thin Airfoil Theory* (Munk, 1922).

5.1 Physical Modelling

The pitching of the airfoil is governed by a time dependent sinusoidal equation that guarantees the oscillation of the angle of attack. As presented in by Berton *et al.* (2002) and McAlister *et al.* (1978) there is generation of a periodic hysteresis cycle when analyzing the aerodynamic coefficients as lift and drag. The governing equations of the selected pitching motion can be described by the following equation.

$$\alpha(t) = \alpha_{min} + \left\{ \left[\frac{\alpha_{max} - \alpha_{min}}{2} \right] [1 - \cos(\omega t)] \right\} \quad (5.1)$$

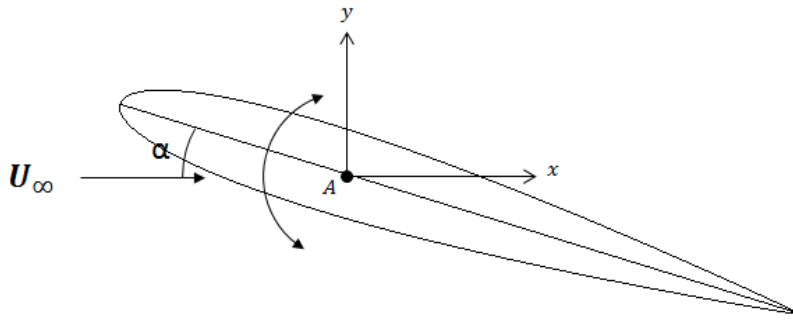


Figure 5.1-1 Schematics for the NACA 0012 airfoil pitch motion

The pitching frequency can be found by the analysis of the oscillation reduced frequency which is a dimensionless parameter. Following the work of McAlister *et al.* (1978), the most significant parameter in the oscillatory motion of an airfoil is its pitching frequency about its

quarter-chord axis. In his work, it is shown that the hysteresis loop enlargement and the dynamic stall recovery is delayed as the reduced frequency, $k = \omega c / 2U_\infty$, is increased; deviating from the static airfoil values for aerodynamic coefficients given as function of the incidence angle.

5.2 Computational Mesh

The mesh used for the calculations is a 69300 nodes quad cell C-Grid topology two-dimensional mesh and was refined to ensure a y^+ less than one for both numerical analyses, using Reynolds number of 10^5 and 2.5×10^6 . The computation domain extent being 20c upstream and 20c downstream the airfoil pitch axis, the inner circular domain has a 10c diameter and is centered at the pitch axis, located at 0.25c aft the leading edge. The mesh uses the Sliding Mesh concept to emulate the pitching motion of the airfoil, the concept is applied to avoid re-meshing and ensure that the cell quality is kept close to the wall.

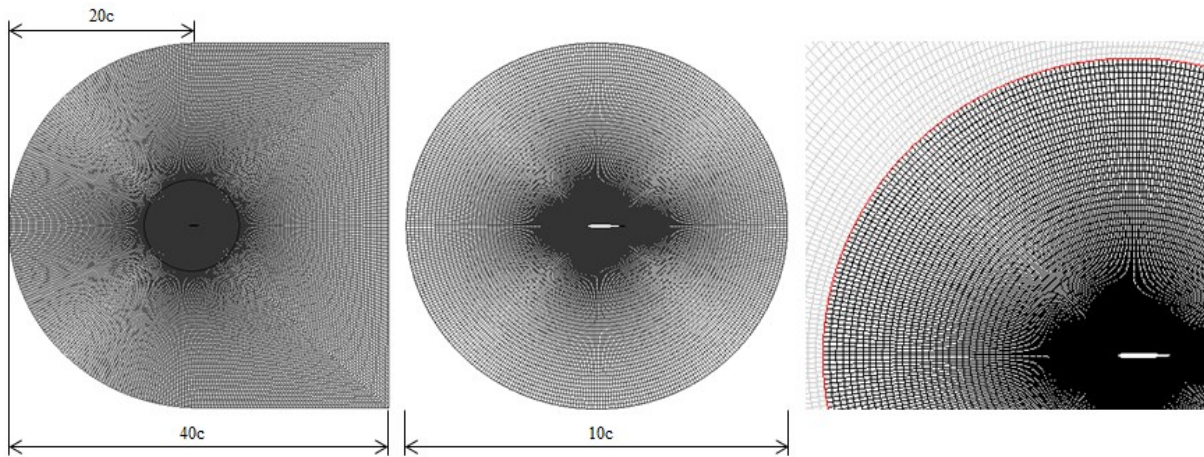


Figure 5.2-1 C-Grid computational domain mesh (left); internal circular domain for sliding mesh (center) and mesh interface domain connection – 'hanging nodes' (right)

The domain is composed by two sub-domains: the external domain, with 100 nodes in I direction and 100 nodes in J direction, with a 75 nodes radial distribution; the internal circular domain, with a 5 chords radial dimension, has 400 nodes distributed in the circumferential direction and 125 nodes in the radial direction. The use of an internal circular domain was to guarantee the mesh movement with respect to the pitching airfoil without changing the quality of

the cells during motion in the ongoing simulation. The grid refinement was performed only with quadrangular elements to achieve numerical stability in the simulations by a high quality mesh. The refinement was sufficient to achieve mesh-independent results.

The concept of Sliding Meshes is applied to the interfaces between the circular and external domain (Fig. 5.2-2), creating non-matching nodes due to the rotation, also known as ‘hanging nodes’. This maintains the accuracy of the flow prediction close to the wall. Since the nodes are supposed to have a steady position in reference to their moving frame, no smoothing dynamic mesh method is necessary and the quality does not change.

In order to avoid conservation problems the connecting walls between the domains are set as interfaces, so the fluid will flow without changes through it. This condition is set to keep the nodes and cell in the inner boundary of the external domain static, while the nodes and cells in the perimeter of the circular domain can slide following the pitching airfoil movement.

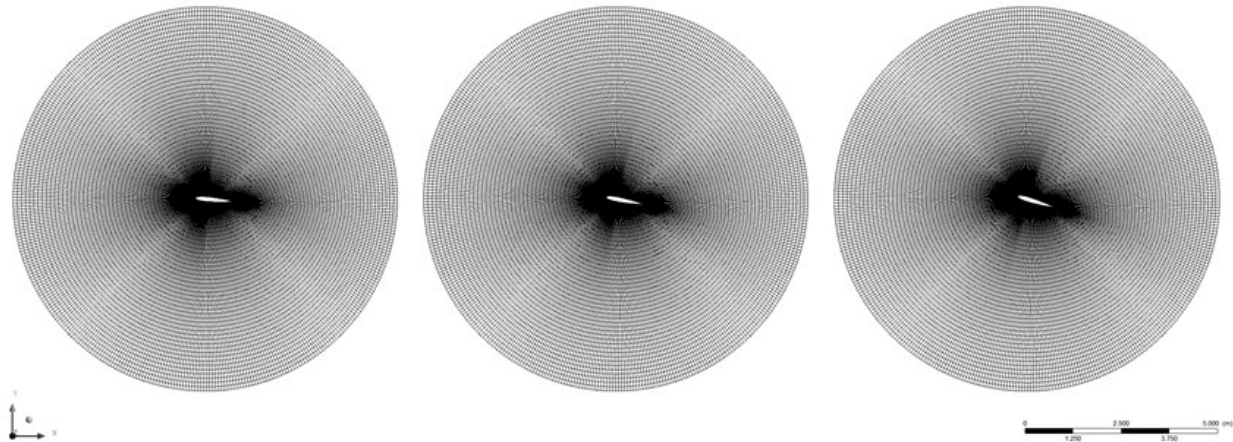


Figure 5.2-2 Sliding Meshes Concept applied to the internal circular domain for pitch motion

5.3 Procedure

For the each analysis case the flow was simulated for 20 pitching cycles and the hysteresis loops where analyzed as an average of the aerodynamic coefficients for the last 5 simulated cycles. The computation was launched from an unsteady simulation of 50s at minimum pitch incidence previous to the airfoil motion. For both analyses the time step was based on the average pitch step, for the first 10 cycles of each case the average pitch step was

$\delta\bar{\alpha}=0.0250^0$; the next 5 cycles had $\delta\bar{\alpha}=0.0125^0$ and the last 5 cycles, $\delta\bar{\alpha}=0.00625^0$. The average pitch angle is defined in Eq. (5.2), where $n_{iterations}$ correspond to the number of iterations per cycle.

$$\delta\bar{\alpha} = \left[\frac{4}{T} \int_0^{T/2} \alpha(t) dt \right] \cdot \frac{1}{n_{iterations}} \quad (5.2)$$

The results analysis for aerodynamics coefficients showed that the computations achieve stability prior to the 10 first cycles; hence for the next 10 cycles there was only a small deviance between each cycle and the averaging. At the inlet of the domain the fluid is flowing in the I direction with a turbulence intensity of 5% and turbulent viscosity ratio of 10. The airfoil walls were set with a no-slip condition and at the outlet the boundary condition was set to zero pressure gradient. Table 5.3-1 summarizes the flow properties for both simulation cases. To model the internal domain pitch an User Defined Function (ANSYS Fluent UDF) was created.

Table 5.3-1 Experiments conditions for the pitch motion and flow properties.

Experiment	Berton <i>et al.</i> (2002)	McAlister <i>et al.</i> (1978)	
Reynolds Number	1.0 x 10 ⁵	2.5 x 10 ⁶	
Minimum Incidence	6 ⁰	5 ⁰	
Maximum Incidence	18 ⁰	25 ⁰	
Reduced Frequency	0.188	0.100	
Airfoil Chord	1.0 m	1.0 m	
Time Stepping	$\delta\bar{\alpha} = 0.025^\circ$	11.89997 x 10 ⁻³ s (960 it.)	5.376734 x 10 ⁻⁴ s (1600 it.)
	$\delta\bar{\alpha} = 0.0125^\circ$	5.949986 x 10 ⁻³ s (1920 it.)	2.688367 x 10 ⁻⁴ s (3200 it.)
	$\delta\bar{\alpha} = 0.00625^\circ$	2.974993 x 10 ⁻³ s (3840 it.)	1.344183 x 10 ⁻⁴ s (6400 it.)
Fluid	Air	Air	
Fluid Density	1.225 kg/m ³	1.225 kg/m ³	
Fluid Viscosity	1.7894 x 10 ⁻⁵ Pa.s	1.7894 x 10 ⁻⁵ Pa.s	
Turbulence Intensity	5%	5%	
Turbulence Viscosity Ratio	10	10	

6 Results and Analysis

The URANS simulations analysis employed the k- ω SST two-dimensional turbulence model to analyze the pitching motion presented on the studies of Berton *et al.* (2002) and McAlister *et al.* (1978). The first set of simulations was based on the Berton *et al.* studies for Reynolds number 10^5 and reduced frequency $k = 0.188$. The oscillation range was 6° to 18° , mean incidence of 12° . The second set of simulation, based on the experiments from McAlister *et al.*, for Reynolds number of 2.5×10^6 and reduced frequency $k = 0.100$. The oscillating range was 5° to 25° , mean incidence of 15° .

As commented in the previous sections, the analysis was based on the averaging of the last 5 cycles of 20 simulated pitching cycles. The time step was based on an average pitch step, which was refined from $\delta\bar{\alpha}=0.0500^\circ$ to $\delta\bar{\alpha}=0.003125^\circ$. For both Reynolds numbers the hysteresis loop presents a refined behavior for an average pitch step of $\delta\bar{\alpha}=0.00625^\circ$. Refining the pitch step beyond that, and consequently the time step, did not affect the behavior any further, and so forth would only result in increasing the computational effort. Hence, the average pitch step of $\delta\bar{\alpha}=0.00625^\circ$ was used for the last 5 cycles average analysis as it is possible to consider the solution time-step independent. The analysis is shown in Fig. 6-1.

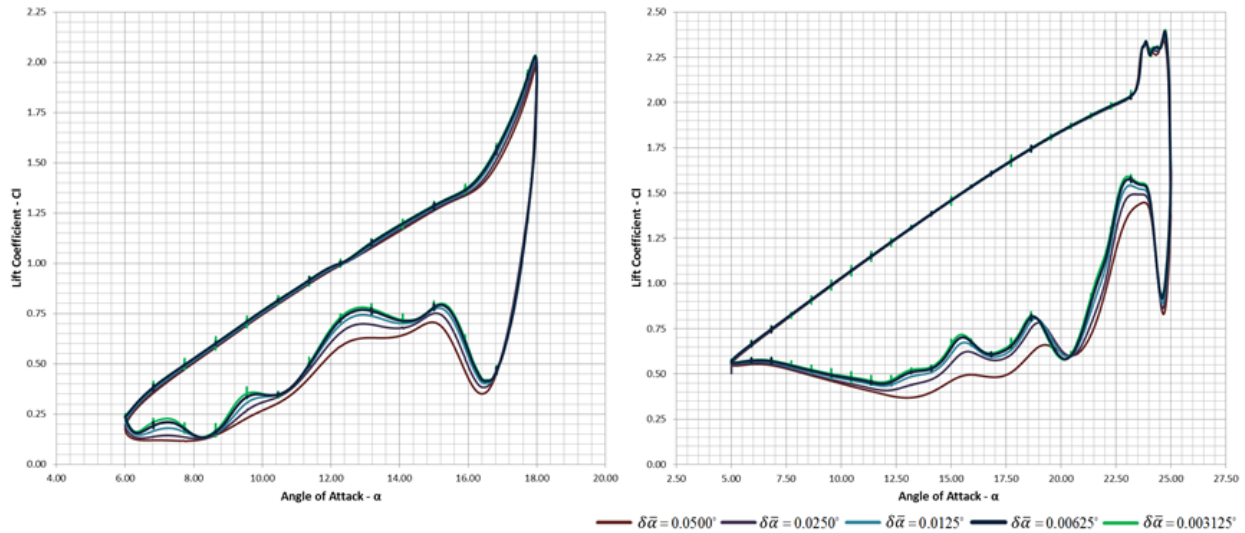


Figure 5.3-1 Analysis of the influence of the average pitch step $\delta\bar{\alpha}$ on the hysteresis loop for lift coefficient at Reynolds 105 and 2.5×10^6 . The average pitch step was refined from $\delta\bar{\alpha}=0.0500^\circ$ to $\delta\bar{\alpha}=0.003125^\circ$

6.1 Pitching Analysis at Reynolds Number 10^5

The simulation results using the $k-\omega$ SST model are compared with the experiments from Berton *et al.* (2002), the two-dimensional $k-\epsilon$ Chien from Martinat *et al.* (2008) and the LES model three-dimensional computations from Kasibhotla & Tafti (2014). However the flow approximation is underestimated for the lift computations during the upstroke phase ($\uparrow$), the SST modelling provides a less critical prediction when compared with the LES during the downstroke ($\downarrow$). Although the SST predicted upstroke phase behavior is underestimated; it is qualitatively close to the LES computation, which indicates that the lift coefficient is not affected by three-dimensional effects.

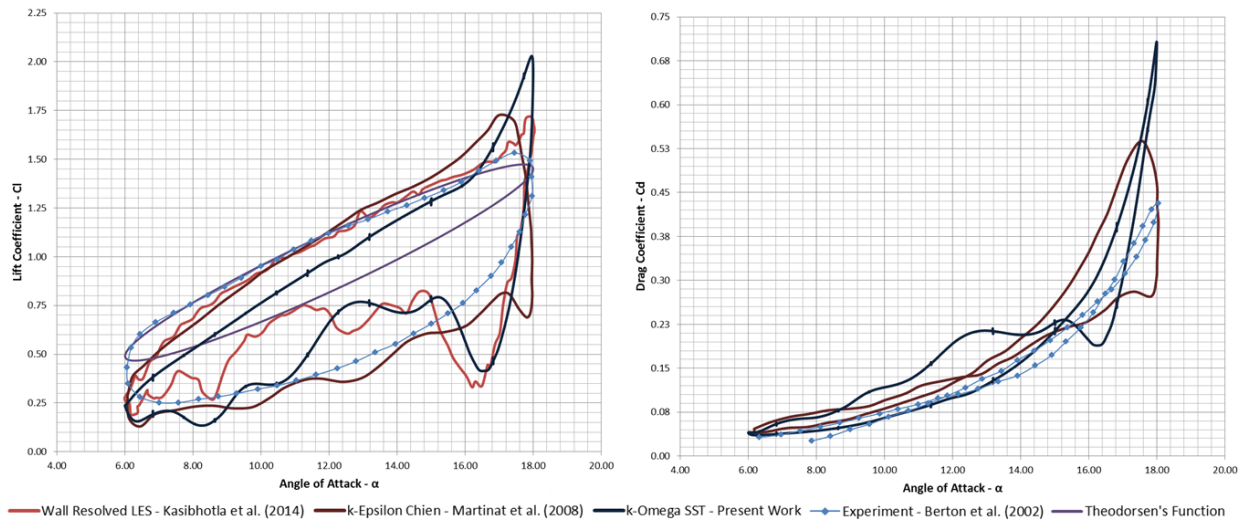


Figure 6.1-1 Hysteresis loops obtained with two-dimensional RANS $k-\omega$ SST analysis for lift and drag compared to experimental results from Berton *et al.* (2002); $k-\epsilon$ Chien turbulence model simulations from Martinat *et al.* (2008) and LES simulations from Kasibhotla & Tafti (2014) and the application of Theodorsen's Function (1934)

The hysteresis loops for the lift coefficient can be seen in Fig. 6.1-1. For the downstroke, the present two-dimensional simulation displays a greater deviation from the experiment. The $k-\epsilon$ Chien analysis from the work by Martinat *et al.* (2008) presents an advance of the lift surge and the boundary layer separation, unlike both 2-D SST and 3-D LES analysis; present a separation delay for the upstroke phase, giving results more optimistic than observed experimentally, as it follows. All the numerical computations overpredict the lift surge; however the lift underestimation during the upstroke is larger in our computations. The numerical predictions

display high oscillation characteristics during the downstroke phase, showing that the model is not able to capture all the circulation of an oscillatory flow with this scale of complexity. Of course, a flow of such complex unsteadiness is not easy to model, as turbulence modelling can render misleading predictions.

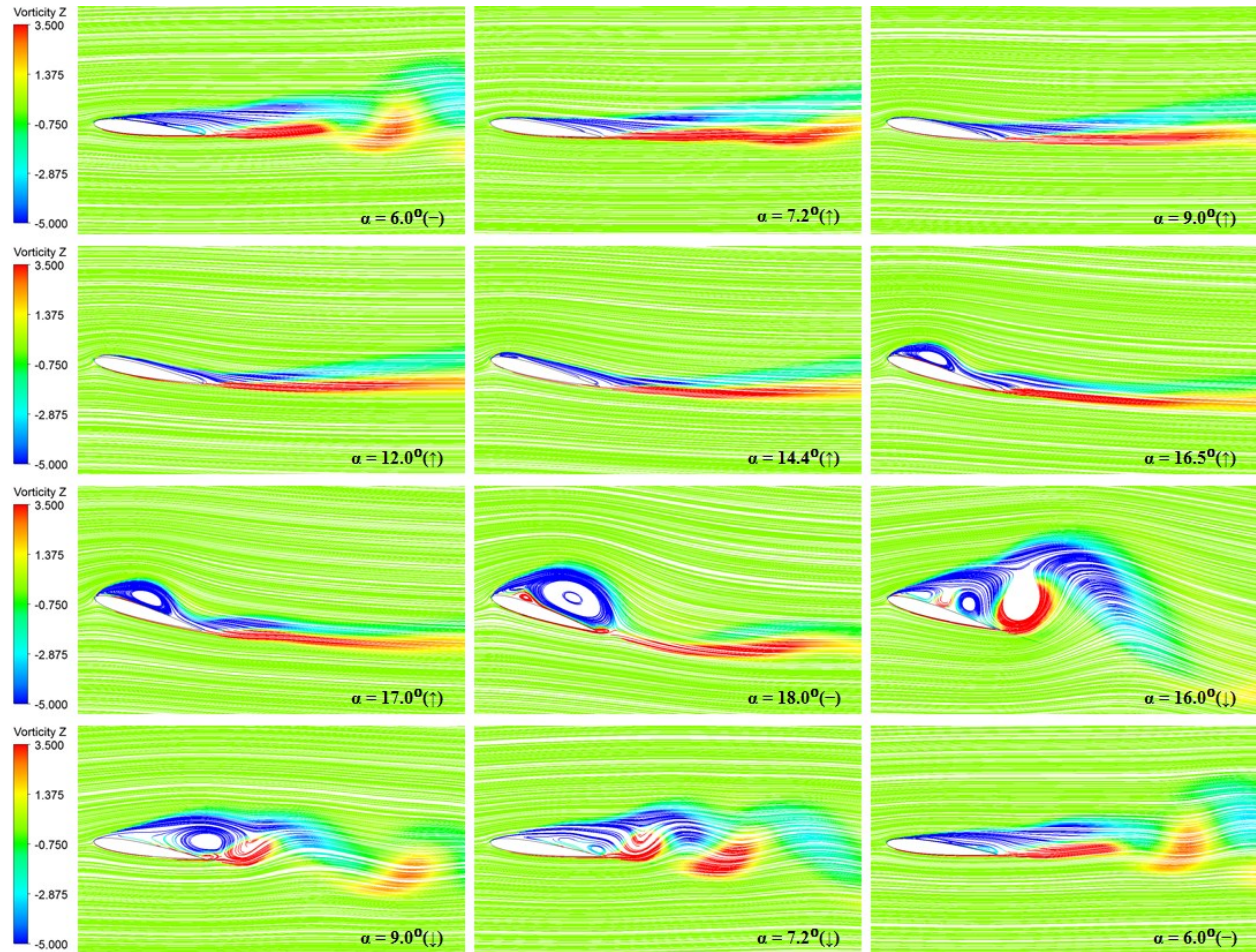


Figure 6.1-2 Vorticity colored streamlines for the NACA0012 pitching airfoil at 10^5 Reynolds number for the $k-\omega$ SST turbulence model, upstroke ($\uparrow$) and downstroke ($\downarrow$) phases

The behavior of the flow for the pitching airfoil is presented in Fig. 6.1-2. During the downstroke phase the main vortex is shed from the surface and many smaller vortices are generated at the airfoil upper surface. As these structures intensities are reduced and carried downstream the airfoil the process of reattachment of the boundary layer starts as the flow is stabilizing close to 7.2° in the downstroke ($\downarrow$). The laminar separation bubble is then visible

close to 6° , when the new pitching cycle begins, and its intensity will be reduced until the flow is fully attached to the upper surface again at 7.2° upstroke ($\uparrow$). The reattachment process is then completed, leading to the latter recirculation zone formation as the vortex shedding displays a periodic behavior, in agreement with the simulations from Kasibhotla & Tafti (2014) and Martinat *et al.* (2008).

It is important to note the influence of the lower surface on the flow behavior during the downstroke phase. As the airfoil pitches down, not only the flow is detaching from the upper surface caused by the vortex shedding, yet the lower surface is acting, in the other direction, producing an upward force and hence reducing the lift. Since the airfoil is symmetric, both surfaces act on the aerodynamic forces generation as explained by the Coanda Effect. Due to its viscosity the fluid will bend around a body as it sticks on the surface (Anderson & Eberhardt, 2001). The difference of speed on the fluid parcels of the boundary layer leads to the creation of shear forces that attach the flow and force it to bend in the direction of the slower layer, which is close to the wall, hence the fluid try to wrap around the object. When the flow is reattached near 7° downstroke ($\downarrow$), the upper surface again bends the air down in the trailing edge, as expected for a positive attitude incidence.

The analysis of the velocity streamlines from the SST computations and the experimental flow velocity field close to the surface of the airfoil surface measured by the Embedded Laser Doppler Velocimetry (ELDV) technique from Berton *et al.* (2002) shows qualitative agreement between both, as observable in Fig. 6.1-3. As expected, due to its flow two-dimensional behavior the upstroke phase is well represented by the $k-\omega$ SST 2D modelling. During the upstroke is possible to observe the formation of the leading edge recirculation region at 14° and at the trailing edge the start of the boundary layer separation, leading to the formation of a vortex that will grow until the airfoil reaches its dynamic stall condition. During the downstroke phase it is possible to observe a slight difference between experimental and computational streamlines, the recirculation region close to leading edge for 14° is similar to the experiments; still, the vortex formation for 10° seems larger for the numerical analysis.

Although there are visible underestimation and overestimation of the aerodynamic coefficients to some extent, considering different incidences of the hysteresis loop, the $k-\omega$ SST computations were capable of capturing accurately the reattachment process of the pitching cycle. Also, in comparison with the experiments, the flow characteristics are in qualitative

agreement; notwithstanding, the overall flow behavior is fairly represented, leading to the conclusion that the SST model can serve as an important tool to the understanding of pitching flow behavior. Also, the loop behavior for the latest SST simulation is by some extent similar to the Wall Resolved LES.

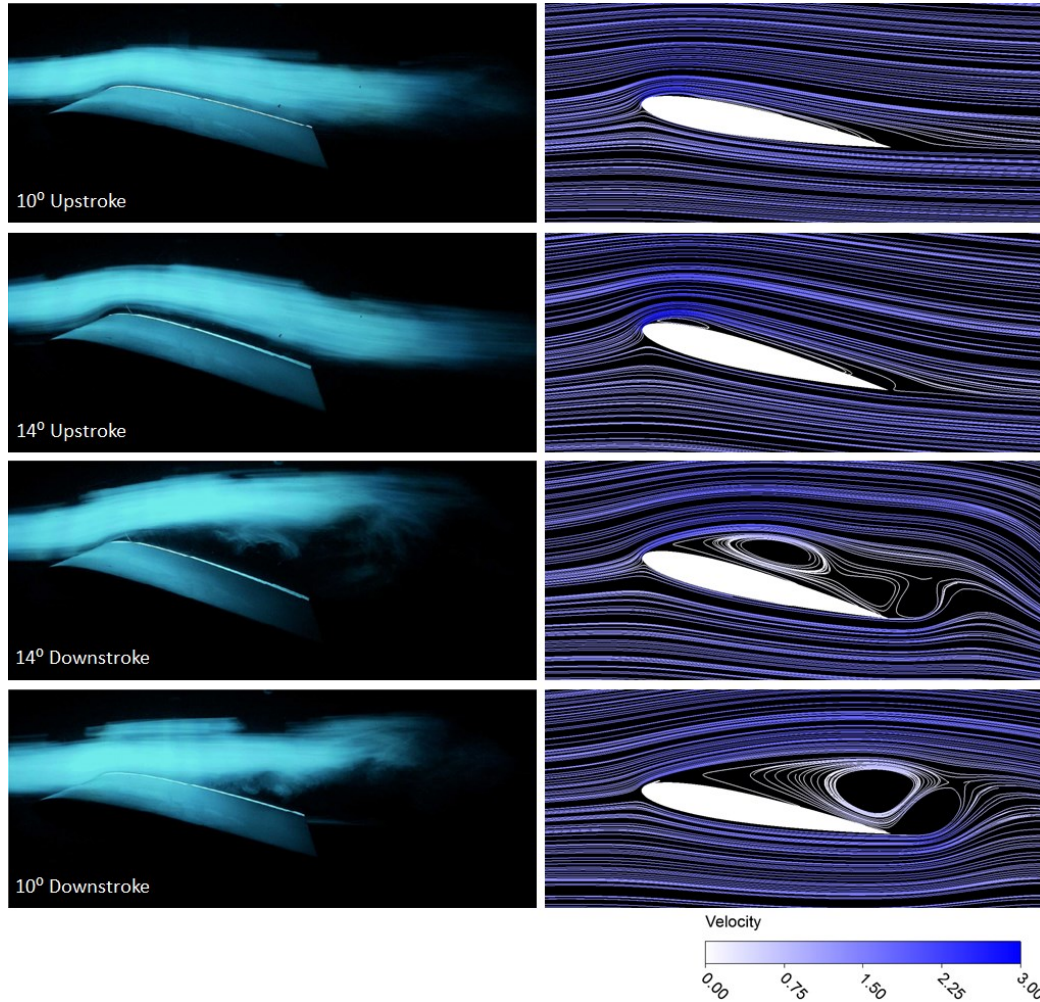


Figure 6.1-3 Analysis of flow behavior for ELDV (Berton *et al.*, 2002) and k - W SST Analysis for 10° and 14° , upstroke ($\uparrow$) and downstroke ($\downarrow$) phases

6.2 Pitching Analysis at Reynolds Number 2.5×10^6

The computational simulation results using the k - ω SST model are compared with the experiments from McAlister *et al.* (1978). The model approximations are closer for Reynolds number of higher order, as can be seen in Fig. 6.2-1, mainly due to the behavior of the boundary

layer, which is expected to be fully turbulent for Reynolds number 2.5×10^6 , thus the $k-\omega$ SST is capable of modelling the flow more precisely. The upstroke phase shows a quite close agreement with experimental data, however, the behavior of the flow for the downstroke phase is still complex to represent, as is possible to see that the lift loss at the beginning of the phase is delayed with respect to experimental data. As seen in the previous analysis, the SST computations also over predict lift, though to a lesser extent.

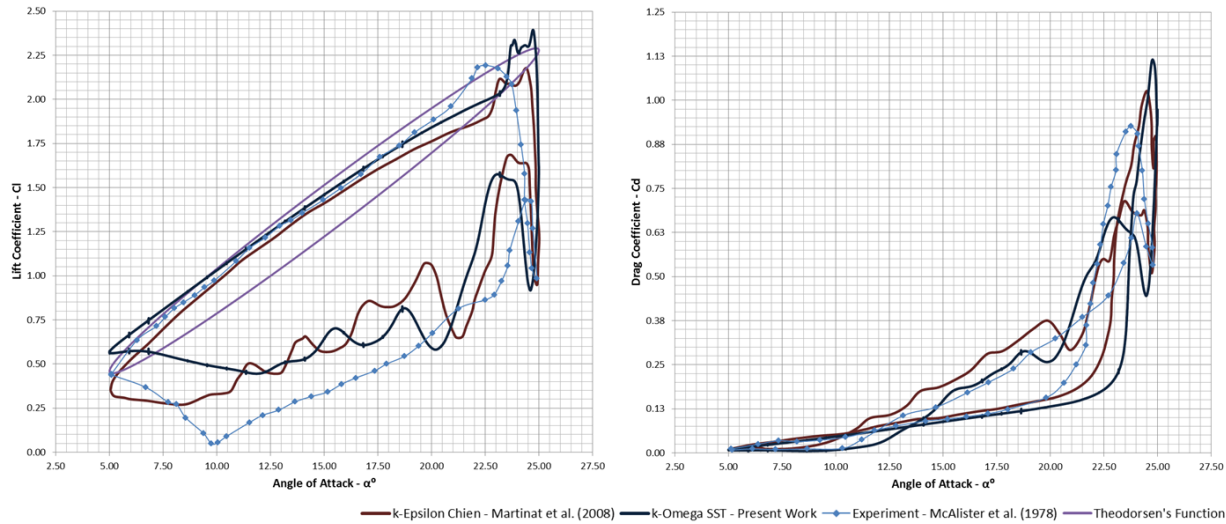


Figure 6.2-1 Hysteresis loops obtained with two-dimensional RANS $k-\omega$ SST analysis for lift and drag coefficients compared to experimental results from McAlister *et al.* (1978) and $k-\epsilon$ Chien turbulence model simulations from Martinat *et al.* (2008) and the application of Theodorsen's Function (1934)

When compared with the $k-\epsilon$ Chien computation from Martinat *et al.* (2008), the surge in the lift coefficient for the simulated $k-\omega$ SST is delayed; however the upstroke phase is more accurate with respect to the experimental analysis for both lift and drag coefficients. During the downstroke it is observable again a high order oscillatory profile, which leads to the same hypothesis from the Reynolds number 10^5 analysis, the flow characteristics is expected to be tridimensional for the phase, contrariwise, due to its almost linear behavior during the upstroke, the flow is fairly represented by the 2-D URANS analysis. Also, the SST model presents a less critical drag coefficient hysteresis loop, even though it overpredicts the drag at the upstroke maximum incidence.

Unlike the previous analysis, as the Reynolds number increases, the flow dynamics change since the boundary layer is accelerated and there is a late formation of the leading edge

recirculation region during the upstroke (Fig. 6.2-2); the trailing edge vortex follows a similar behavior since it starts its growth at high incidence angles and then moves upstream the wall, reaching its maximum intensity until it sheds away with the flow in the start of the downstroke phase.

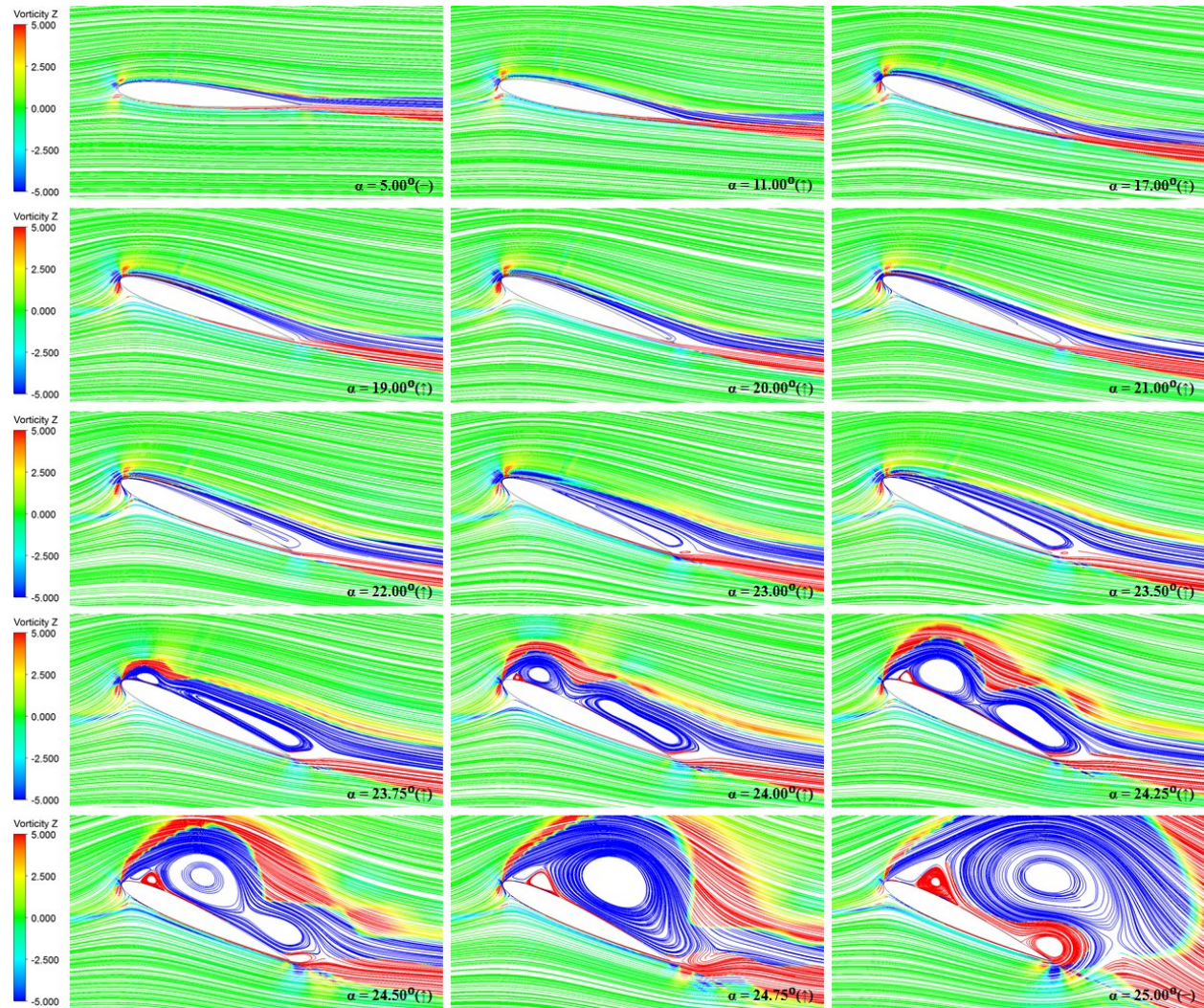


Figure 6.2-2 Vorticity colored streamlines for the NACA0012 pitching airfoil at 2.5×10^6 Reynolds number for the $k-\omega$ SST turbulence model, upstroke phase ($\uparrow$)

During the upstroke the boundary layer remains attached to the wall up to approximately $17^\circ(\uparrow)$, where the flow starts to detach from the trailing edge and leads to the beginning of the reversed flow at $20^\circ(\uparrow)$. At this incidence, the difference between the experimental results from McAlister *et al.* (1978) can be noticed, in which the surge in the lift coefficient will start and the

vortex shedding from the surface will lead to the dynamic stall before the maximum incidence. However, in the $k-\omega$ SST numerical analysis the trailing edge vortex starts from $20^\circ(\uparrow)$, reaching the leading vortex up to $24^\circ(\uparrow)$. From $24^\circ(\uparrow)$ to 25° smaller vortical structures begin to form as well as the main structure starts to shed from the wall and the airfoil reaches its dynamic stall condition. As for the downstroke phase (Fig. 6.2-3), it is possible to note how the flow interacts with the trailing edge vortex and also with the airfoil lower surface since the flow is bent upwards, showing again the importance of the Coanda Effect to the viscous fluid flow analysis. The airfoil remains in the dynamic stall condition mostly during all the downstroke, until approximately $12^\circ(\downarrow)$.

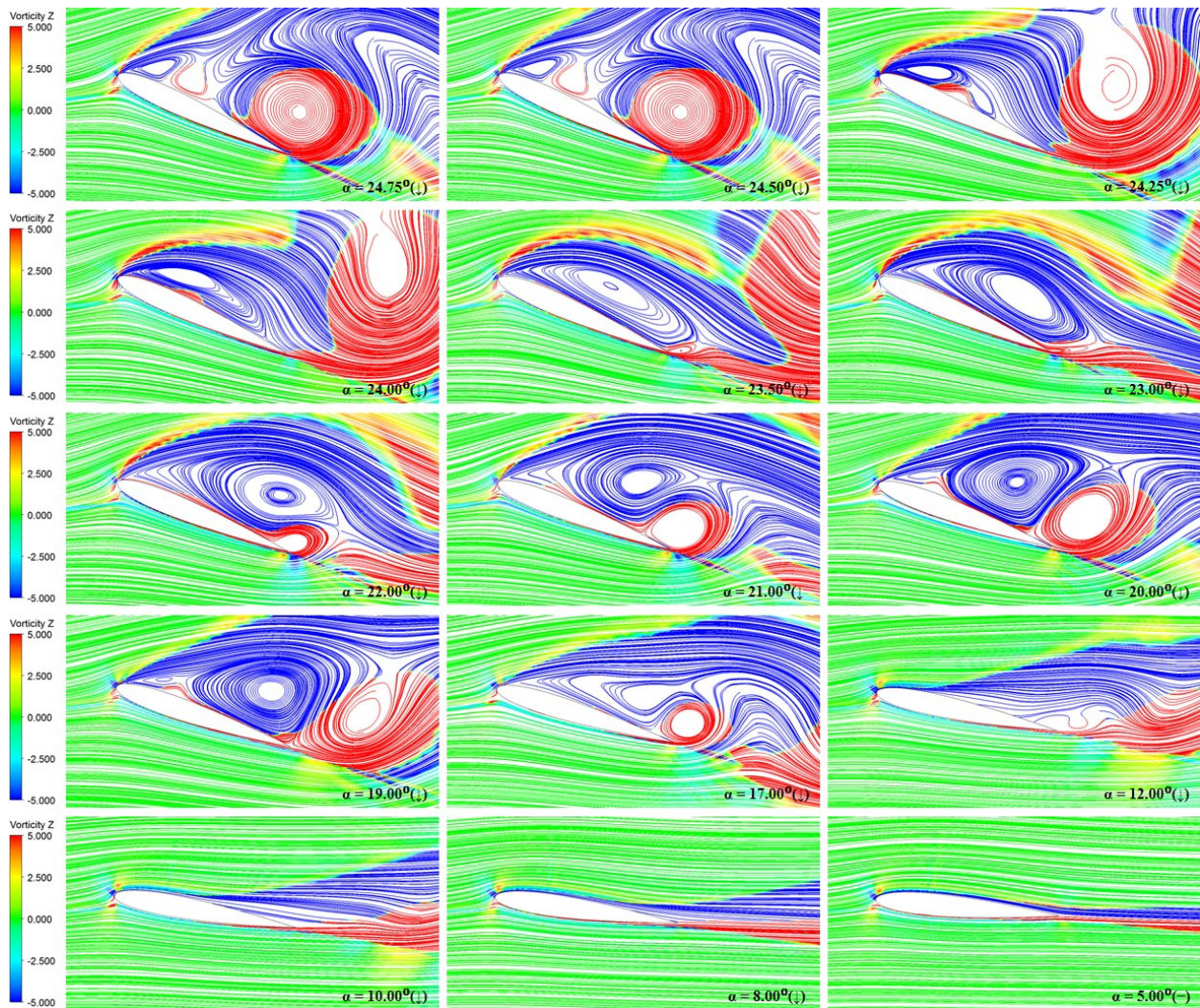


Figure 6.2-3 Vorticity colored streamlines for the NACA0012 pitching airfoil at 2.5×10^6 Reynolds number for the $k-\omega$ SST turbulence model, downstroke phase ($\downarrow$)

At $11^\circ(\downarrow)$ the vortical structures are shed away in the flow and the boundary layer reattachment process begins close to $8^\circ(\downarrow)$. This is in agreement with the experiment results, where it is possible to observe that the dynamic stall remains until $10^\circ(\downarrow)$. Up to 5° the airfoil regains lift due to the boundary layer reattachment to the surface. From $6^\circ(\downarrow)$ to 5° the flow is already fully attached to the surface, completing the pitching cycle. It is important to notice that, since the flow is more accelerated, the boundary layer reattachment process can occur more rapidly, still during the downstroke phase.

The analysis of the aerodynamic coefficients and streamlines for the pitching cycle has shown that the $k-\omega$ SST turbulence model capabilities on modelling unsteady flows improve as the Reynolds number increases. Although there are differences from the experiments, the flow is better represented at Reynolds 2.5×10^6 and the 2-D URANS vortex shedding analysis can render important insights at low computational cost on the understanding of pitching airfoils. It is worth noticing that as the Reynolds number increases, the flow is closer to the potential approximation for the upstroke phase, as the velocity field is nearly irrotational up to $17^\circ(\uparrow)$, as can be seen in Fig. 6.2-4. The boundary layer is highly energized due to the flow acceleration in the leading edge and it is kept thin and close to the surface. It is possible to treat the flow as inviscid and irrotational, since viscous effects are limited to the boundary layer.

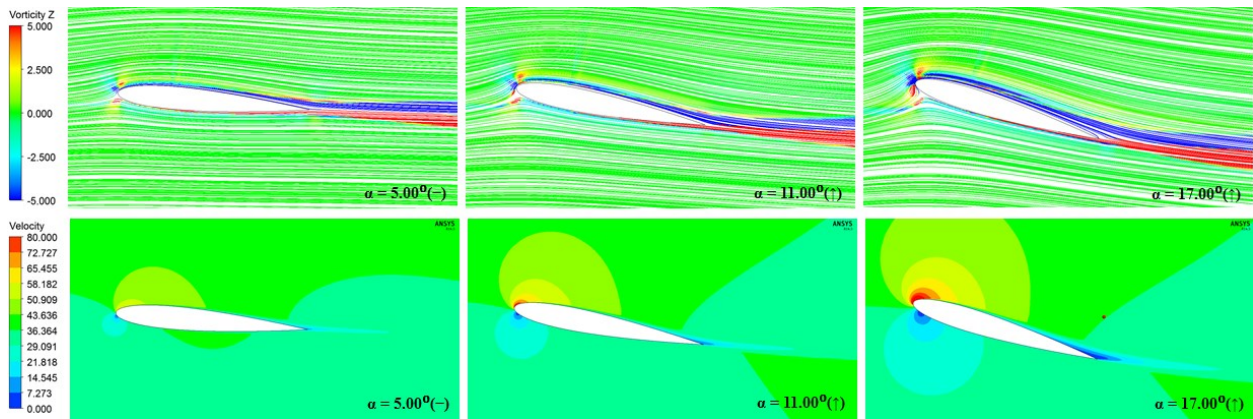


Figure 6.2-4 Vorticity colored streamlines and velocity contour (0 to 80 m/s) for the NACA0012 pitching airfoil at 2.5×10^6 Reynolds number for the $k-\omega$ SST turbulence model; 5° , 11° and 17° , upstroke phase ($\uparrow$). Local maximum velocity of 61.39, 78.71 and 113.67 m/s and global maximum velocity of 121.52 m/s

The modeling of flow as a potential was addressed to the Theodorsen's Theory (1934) as well as in the studies by McCroskey (1982). The same assumption is made by Yang *et al.* (2006)

when using the Euler Method as a tool for the analysis of unstalled pitching airfoils. However, as shown in the previous analyses, this assumption is only valid for the upstroke phase before the generation of the leading and trailing edge vortices and the recirculation region close to the wall. As the vortex sheds, the flow becomes highly rotational and the Potential Flow Theory is no longer applicable.

From the analysis of the lift coefficient hysteresis loops for the Berton *et al.* (2002) and McAlister *et al.* (1978) experiments in comparison with Theodorsen's function, as shown respectively by Fig. 6.1-1 and Fig. 6.2-1, it is possible to observe that the Potential Theory can be addressed with fair results for the upstroke phase. Although the Theodorsen's Theory (1934) has its limitations, the upstroke phase matched the experimental results for low order Reynolds number, as the flow can be modelled as potential as shown by Fig. 6.2-4. The surge of lift encountered seen in both experimental and numerical results cannot be accounted by the Theodorsen's Theory as well, as it is a consequence of the vortex residence at the airfoil upper surface at the end of upstroke phase, presenting a high energy profile due to flow acceleration and vorticity. As discussed, the downstroke phase has a highly rotational behavior as well, which is not accounted by the Theodorsen's function; hence the hysteresis loops for the theory cannot model the flow, differing from the experimental and numerical results.

The comparison between the experiments and both URANS computations, the $k-\omega$ SST and the $k-\epsilon$ Chien from Martinat *et al.* (2008), shows that the method can serve as an important tool for the unsteady analysis, rendering fair results with less computational effort than an LES, for instance. Both cases displayed misleading results in a minor extent, which is expected due to the two-dimensional approach.

7 Conclusions

The $k-\omega$ SST model is capable of predicting the behavior of the flow around pitching airfoils, also considering that the mesh is sufficiently refined for achieving such results. In the computations for the Berton *et al.* case the difference between simulations and experiments is mainly due to the transitional behavior of the boundary layer. Consequently, the SST model is likely to face problems on the modeling of such flows. Although the LES simulations from Kasibhotla & Tafti (2014) show a better agreement with the upstroke phase, the SST simulations display a downstroke less deviant from the experiments. Qualitatively, the SST presents a loop that is close to the LES behavior, showing that the SST model can provide results of reasonable quality while saving computational effort.

The analysis of the McAlister *et al.* computational case hysteresis loop shows better agreement between numerical computations and experimental data. Although there is clearly an overprediction of lift and delay on the dynamic stall angle prediction, the hysteresis loop is qualitatively close to the experiment. It was also possible to confirm the quasi potential behavior during most of the upstroke phase.

The complexity of the vortex shedding during the downstroke is not well modeled by the two-dimensional URANS, since it strongly three-dimensional behavior is inherent. One can conclude that the $k-$ SST is capable of predicting some of the characteristics of pitching flows, not yet fully, nevertheless leading to important insights, as the qualitative behavior matched the expected one.

The use of a Sliding Mesh concept is proven to be reliable for steady flows, as the comparison with other results cases from different turbulence models have shown similarity on flow behavior and results. The mesh refining close to the near-wall region is crucial for achieving an accurate modelling, and the use of a sliding mesh for unsteady flows simulation can assure results as reliable as the usual hybrid tri/quad dynamic meshes, since it is possible to maintain the mesh quality at near-wall regions for quad cells during the mesh motion.

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Appendix

APPENDIX A – MATLAB Routines for the Analysis on Pitching Airfoils

This appendix provides the necessary MATLAB routines for the analysis of the pitching airfoil hysteresis loops. The routines and subroutines are presented as it follows.

```
% ----- %
% THE STUDY OF DYNAMIC STALL AND URANS CAPABILITIES ON MODELLING PITCHING
% AIRFOIL FLOWS
% ----- %

% Federal University of Uberlandia
% Faculty of Mechanical Engineering
% Bachelor on Aeronautical Engineering

% June 14th, 2015
% Routine for the Analysis of the Pitching Hysteresis Loop variation with
% respect to the reduced frequency and pitching axis.

% Alexandre Felipe Medina Correa
% Advisor: Prof. Dr. Francisco José de Souza.

%% Routine DynamicPitching.m
clear all
close all
clc
%% Geometric Parameters
% Airfoil Chord [m]
c = 1;
% Half Chord [m]
b = c/2;
%% Flow Parameters
% Free-stream velocity [m/s]
U = 1.46073469;
% Air Density [kg/m3]
rho_air = 1.225;
%% Pitching Motion Parameters
% Minimum Incidence
alpha_min = 6*(pi/180);
% Maximum Incidence
alpha_max = 18*(pi/180);
% Mean Incidence
alpha_bar = (alpha_max - alpha_min)/2;

%% Theodorsen's Function for Pitching Motion - Reduced Frequency
% Pitch Axis Position from Leading Edge [m]
A = c/4;
% Reduced Frequency
k_1 = 0.100; k_2 = 0.188; k_3 = 0.376;
% Average Angular Frequency [rad/s]
wo_1 = 2*U*k_1/c;
wo_2 = 2*U*k_2/c;
```

```

wo_3 = 2*U*k_3/c;
% Pitch Period
T_1 = 2*pi/wo_1;
T_2 = 2*pi/wo_2;
T_3 = 2*pi/wo_3;
% Time Parameters
t_1 = 0;
dt_1 = T_1/5000;
t_2 = 0;
dt_2 = T_2/5000;
t_3 = 0;
dt_3 = T_3/5000;

for i = 1:5001
    % Angular Position, Velocity and Acceleration
    alpha_1(i) = alpha_min + alpha_bar*(1 - cos(wo_1*t_1));
    alpha_d_1(i) = alpha_bar*wo_1*sin(wo_1*t_1);
    alpha_d2_1(i) = alpha_bar*(wo_1^2)*cos(wo_1*t_1);
    alpha_2(i) = alpha_min + alpha_bar*(1 - cos(wo_2*t_2));
    alpha_d_2(i) = alpha_bar*wo_2*sin(wo_2*t_2);
    alpha_d2_2(i) = alpha_bar*(wo_2^2)*cos(wo_2*t_2);
    alpha_3(i) = alpha_min + alpha_bar*(1 - cos(wo_3*t_3));
    alpha_d_3(i) = alpha_bar*wo_3*sin(wo_3*t_3);
    alpha_d2_3(i) = alpha_bar*(wo_3^2)*cos(wo_3*t_3);
    % Lift Coefficient
    coeff_1_1 = (U*alpha_d_1(i) + alpha_d2_1(i)*(b - A))*b/U;
    coeff_2_1 = (U*alpha_1(i) + alpha_d_1(i)*(3*b/2 - A))*2*theod(k_1);
    CL_1(i) = (coeff_1_1 + coeff_2_1)*(pi/U);
    coeff_1_2 = (U*alpha_d_2(i) + alpha_d2_2(i)*(b - A))*b/U;
    coeff_2_2 = (U*alpha_2(i) + alpha_d_2(i)*(3*b/2 - A))*2*theod(k_2);
    CL_2(i) = (coeff_1_2 + coeff_2_2)*(pi/U);
    coeff_1_3 = (U*alpha_d_3(i) + alpha_d2_3(i)*(b - A))*b/U;
    coeff_2_3 = (U*alpha_3(i) + alpha_d_3(i)*(3*b/2 - A))*2*theod(k_3);
    CL_3(i) = (coeff_1_3 + coeff_2_3)*(pi/U);
    % Time Step
    t_1 = t_1 + dt_1; t_2 = t_2 + dt_2; t_3 = t_3 + dt_3;
end
alpha_1 = alpha_1*(180/pi);
alpha_2 = alpha_2*(180/pi);
alpha_3 = alpha_3*(180/pi);
subplot(1,2,1)
plot(alpha_1,CL_1,'b-','linewidth',1.50);
hold on
subplot(1,2,1)
plot(alpha_2,CL_2,'r-','linewidth',1.50);
hold on
subplot(1,2,1)
plot(alpha_3,CL_3,'k-','linewidth',1.50);
grid on
hold off
xlabel('Angle of Attack (\alpha)');
ylabel('Lift Coefficient - Cl');
legend('k=0.100',...
       'k=0.188',...
       'k=0.376');
%% Theodorsen's Function for Pitching Motion - Pitch Axis
% Pitch Axis Position from Leading Edge [m]

```

```

A_4 = c/4; A_5 = c/2; A_6 = 3*c/4;
% Reduced Frequency
k = 0.200;
% Average Angular Frequency [rad/s]
wo = 2*U*k/c;
% Pitch Period
T = 2*pi/wo;
% Time Parameters
t = 0;
dt = T/5000;
for i = 1:5001
    % Angular Position, Velocity and Acceleration
    alpha(i) = alpha_min + alpha_bar*(1 - cos(wo*t));
    alpha_d(i) = alpha_bar*wo*sin(wo*t);
    alpha_d2(i) = alpha_bar*(wo^2)*cos(wo*t);
    % Lift Coefficient
    coeff_1_4 = (U*alpha_d(i) + alpha_d2(i)*(b - A_4))*b/U;
    coeff_2_4 = (U*alpha(i) + alpha_d(i)*(3*b/2 - A_4))*2*theod(k);
    CL_4(i) = (coeff_1_4 + coeff_2_4)*(pi/U);
    coeff_1_5 = (U*alpha_d(i) + alpha_d2(i)*(b - A_5))*b/U;
    coeff_2_5 = (U*alpha(i) + alpha_d(i)*(3*b/2 - A_5))*2*theod(k);
    CL_5(i) = (coeff_1_5 + coeff_2_5)*(pi/U);
    coeff_1_6 = (U*alpha_d(i) + alpha_d2(i)*(b - A_6))*b/U;
    coeff_2_6 = (U*alpha(i) + alpha_d(i)*(3*b/2 - A_6))*2*theod(k);
    CL_6(i) = (coeff_1_6 + coeff_2_6)*(pi/U);
    % Time Step
    t = t + dt;
end
alpha = alpha*(180/pi);
subplot(1,2,2)
plot(alpha,CL_4,'b-','linewidth',1.50);
hold on
subplot(1,2,2)
plot(alpha,CL_5,'r-','linewidth',1.50);
hold on
subplot(1,2,2)
plot(alpha,CL_6,'k-','linewidth',1.50);
grid on
hold off
xlabel('Angle of Attack (\alpha)');
ylabel('Lift Coefficient - Cl');
legend('A=0.25c',...
       'A=0.50c',...
       'A=0.75c');
%% END of Routine

```

```

% ----- %
% THE STUDY OF DYNAMIC STALL AND URANS CAPABILITIES ON MODELLING PITCHING
% AIRFOIL FLOWS
% ----- %

% Federal University of Uberlandia
% Faculty of Mechanical Engineering
% Bachelor on Aeronautical Engineering

% Alexandre Felipe Medina Correa
% Advisor: Prof. Dr. Francisco José de Souza.

% ----- %

% May 11th, 2015
% Matlab Function to Calculate the Modified Bessel Functions for
% Theodorsen's Function.
% From Notes on Aircraft Loads and Aeroelasticity.
% Prof. MSc. Thiago Augusto Machado Guimarães

%% Function theod.m
function thd = theod(rk)
%% Quasi-Steady Approach
if (rk == 0)
    thd = 1;
%% Unsteady Approach
else
    i = sqrt(-1);
    thd = besselk(1,i*rk)/(besselk(0,i*rk)+besselk(1,i*rk));
end
%% END of Function Routine

```

APPENDIX B – Fluent User Defined Function (UDF) for Modelling the Pitching Motion

This appendix provides the necessary Fluent UDF codes for the analysis of the pitching airfoil hysteresis loops simulation for sliding meshes. The C routines are presented as it follows.

```

/*****
      NACA0012 Rotation UDF - Berton Case

Federal University of Uberlandia
Faculty of Mechanical Engineering
Bachelor on Aeronautical Engineering

AUG 2014
UDF for modelling the pitching motion for a mesh domain, considering the
harmonic pitch of a NACA 0012 airfoil equation.

Alexandre Felipe Medina Correa
Advisor: Prof. Dr. Francisco José de Souza.
*****/

#include "udf.h"
#include "dynamesh_tools.h"

/* Definition of Pi */
#define pi acos(-1.)
/* Initial Time Definition */
#define Start 50.0

DEFINE_CG_MOTION(airfoil, dt, vel, omega, time, dtime)
{

    /***
    Declaration of Variables ***/

    float wo, alpha, t, T, Stop, amin, amax;

    /* Wave Angular Velocity (rad/s) */
    wo = 0.550;

    /* Half Oscillation Deflection (rad) - 12 degrees */
    alpha = 12*(pi/180);

    /* Time Parameters */
    /* Current Time */
    t = time;
    /* Oscillation Period */
    T = 2*pi/wo;
    /* Stop Time - Dependent on the Number of Oscilations */
    Stop = (25*T + Start);

```

```

/* Angular Motion Data */
    /* Minimum Deflection */
    amin = (6)*(pi/180);
    /* Maximum Deflection */
    amax = (18)*(pi/180);

/*****
/***** Angular Velocity Calculation *****/
/*****/

/* For anytime less than 2s, the velocity remains constant */
if (t <= Start)
{ omega[2] = 0; }
else
{
    /* For any time between Start [s] and Stop [s], then,
    the angular velocity will vary as function of time in
    order to simulate an oscillatory movement. */

    /* Motion Angular Velocity */
    if (t > Start && t <= Stop )
    { omega[2] = ((0.5*wo*(amin - amax))*sin(wo*(t-Start))); }
    /* Stop Condition */
    else
    {
        /* After completeting the motion, the angular velocity
        returns to 0 rad/s. */
        if (t > Stop)
        { omega[2] = 0; }
    }
}
}

/*****
NACA0012 Rotation UDF - McAlister Case

Federal University of Uberlandia
Faculty of Mechanical Engineering
Bachelor on Aeronautical Engineering

AUG 2014
UDF for modelling the pitching motion for a mesh domain, considering the
harmonic pitch of a NACA 0012 airfoil equation.

Alexandre Felipe Medina Correa
Advisor: Prof. Dr. Francisco José de Souza.
*****/

#include "udf.h"
#include "dynamesh_tools.h"

/* Definition of Pi */
#define pi acos(-1.)
/* Initial Time Definition */
#define Start 50.0

```

```

DEFINE_CG_MOTION(airfoil, dt, vel, omega, time, dtime)
{

    /*****
    /***** Declaration of Variables *****/
    /*****/

    float wo, alpha, t, T, Stop, amin, amax;

    /* Wave Angular Velocity (rad/s) */
    wo = 7.30367345;

    /* Half Oscillation Deflection (rad) - 15 degrees */
    alpha = 15*(pi/180);

    /* Time Parameters */
        /* Current Time */
        t = time;
        /* Oscillation Period */
        T = 2*pi/wo;
        /* Stop Time - Dependent on the Number of Oscilations */
        Stop = (25*T + Start);

    /* Angular Motion Data */
        /* Minimum Deflection */
        amin = (5)*(pi/180);
        /* Maximum Deflection */
        amax = (25)*(pi/180);

    /*****
    /***** Angular Velocity Calculation *****/
    /*****/

    /* For anytime less than 2s, the velocity remains constant */
    if (t <= Start)
    { omega[2] = 0; }
    else
    {
        /* For any time between Start [s] and Stop [s], then,
        the angular velocity will vary as function of time in
        order to simulate an oscillatory movement. */

        /* Motion Angular Velocity */
        if (t > Start && t <= Stop )
        { omega[2] = ((0.5*wo*(amin - amax))*sin(wo*(t-Start))); }
        /* Stop Condition */
        else
        {
            /* After completeting the motion, the angular velocity
            returns to 0 rad/s. */
            if (t > Stop)
            { omega[2] = 0; }
        }
    }
}
}

```