

Figure 4.14: A-series. Influence of the domain size on the relative position at the end of the simulation. Left to right: 4ϕ , 6ϕ , 8ϕ , 10ϕ .

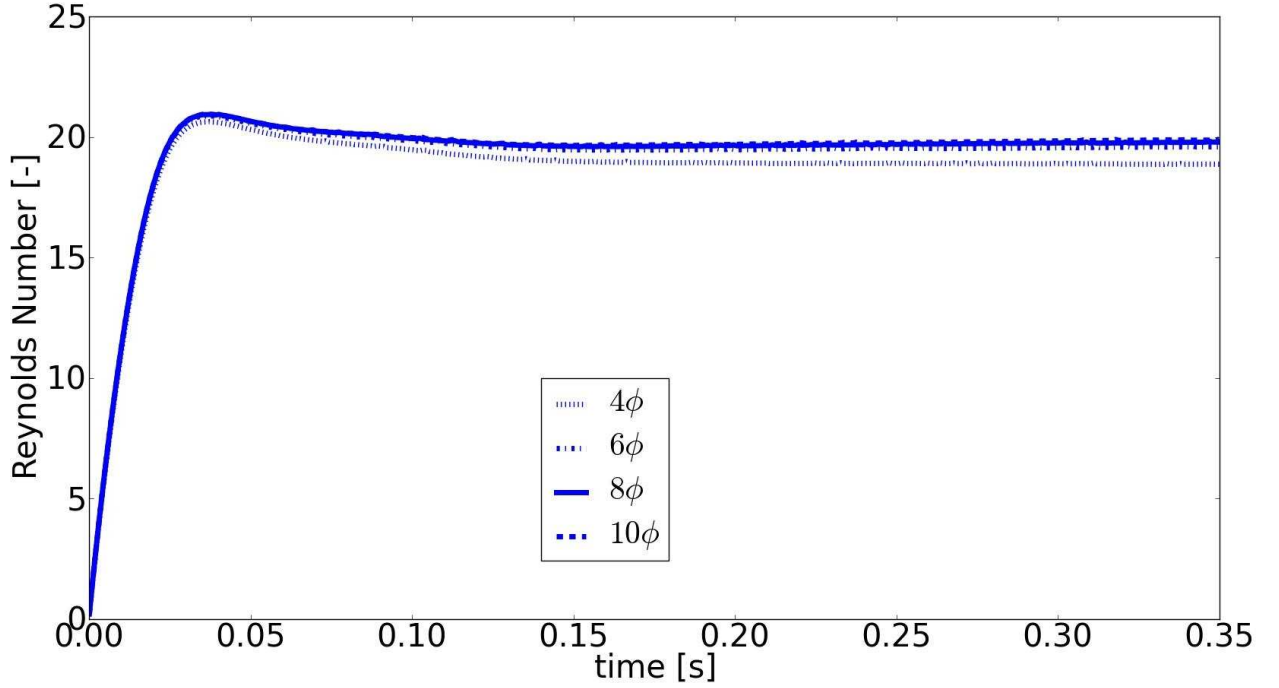


Figure 4.15: A-series. Influence of the domain size on the bubble Re profile. Reference Reynolds number $Re = 20.6$.

Characteristic length of the Lagrangian Mesh

The influence of the characteristic length of the Lagrangian mesh was studied by simulating the test case in three different configurations. The same number of refinement levels and grid spacing was used on all simulations. The bubble interface was discretized with 3 successively refined meshes and all other simulation parameters were kept constant. The length ratio is defined as $\lambda_\xi = \frac{\xi_E}{\xi_L}$, where ξ_E is the characteristic length of the Eulerian grid, defined at the level containing the Lagrangian mesh, and ξ_L is the mean edge length of the Lagrangian mesh.

Figure (4.16) compares the mesh resolution for the three cases simulated and the time history of the Reynolds profile is shown in Figure (4.17). The value of the error based on the terminal Re is shown in Table (4.8), confirming the small difference between them. Proximity in the results is also seen in Fig. (4.18), which shows the effect of the mesh resolution on the representation of

high curvature regions.

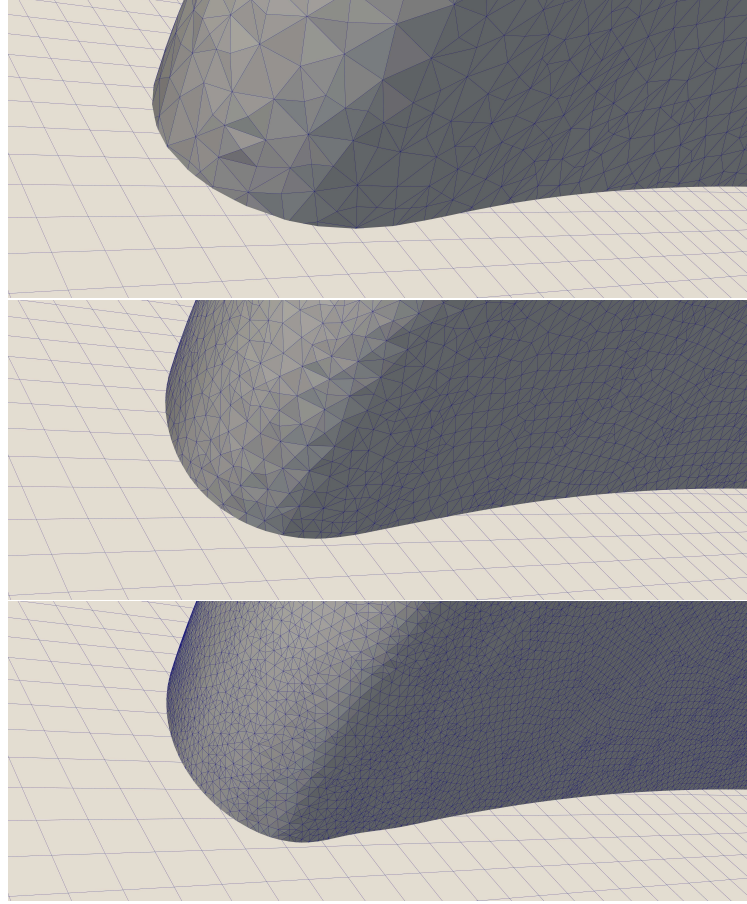


Figure 4.16: Lagrangian mesh resolution. From top to bottom: $\lambda_\xi = 1$, $\lambda_\xi = 2$ and $\lambda_\xi = 4$

Table 4.8: Influence of the Lagrangian mesh resolution on the terminal bubble Reynolds number. Reference $Re = 20.6$.

Case	λ_ξ	Re	Error (%)
A9	1	19.707	4.333
A10	2	19.826	3.757
A11	4	19.803	3.869

Density Ratio

The influence of the density ratio on the bubble ascending velocity was studied for three different bubble regimes. The first case simulated the rising of an oblate ellipsoidal bubble defined by $Eo = 243$, $M = 266$ and $Re = 7.77$. This case was based on the work of Bhaga (1976), who employed a density ratio $\lambda_\rho = 1370$ in his experiment. Three simulations were performed, for density ratios $\lambda_\rho = 10, 100, 1000$. Figure (4.19) shows the Reynolds profile for each case simulated, while

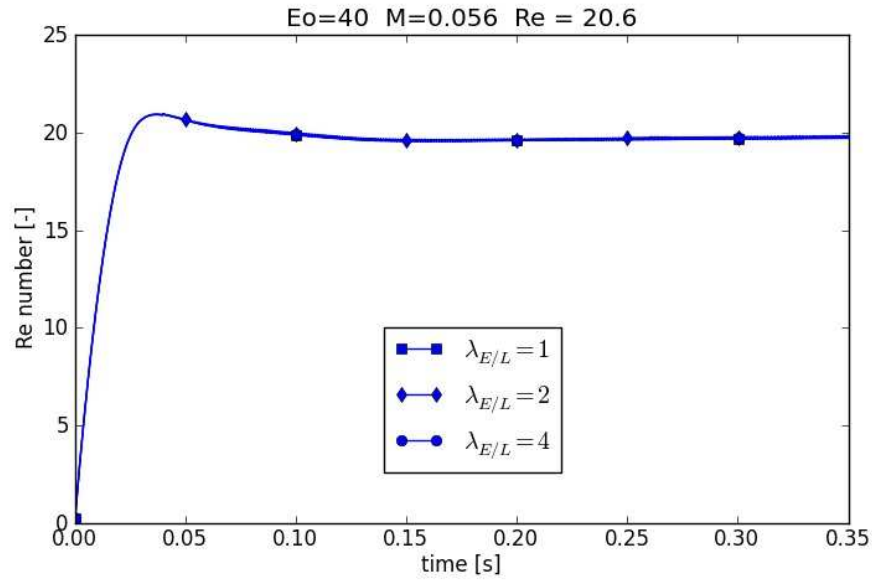


Figure 4.17: Influence of the Lagrangian mesh resolution on the bubble Reynolds profile.



Figure 4.18: Cross section of the terminal bubble shape depending on the Lagrangian edge length, for a fixed Eulerian grid size and varying Lagrangian characteristic length. $\circ$: $\lambda_\xi = 1$. $\square$: $\lambda_\xi = 2$. $\triangle$: $\lambda_\xi = 4$.

Table (4.9) shows the terminal shape and Re number. As expected, the error on the terminal Re number decreases as the density ratio approaches the experimental value. However, the average time cost for solving one time step in $\lambda_p = 100$ increases 50% when compared to $\lambda_p = 10$, and so does $\lambda_p = 1000$ when compared to $\lambda_p = 100$.

The second case simulated was a bubble rising in the creeping flow regime. Results are again

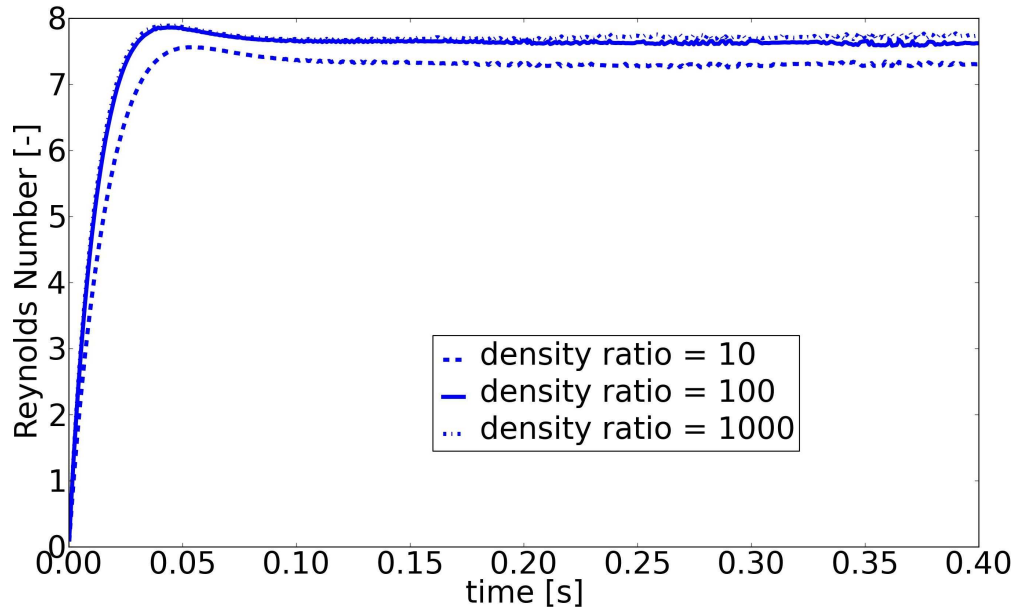
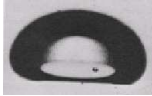


Figure 4.19: Rising bubble. Influence of the density ratio on the ascending velocity.

Table 4.9: Influence of the density ratio on the terminal Reynolds number and bubble shape. Comparison against the experimental work of Bhaga and Weber (1981). $Eo = 243$, $M = 266$, $Re = 7.77$.

Test Case	Shape	Re	Error (ϵ [%])
Experiment (BHAGA; WEBER, 1981)		7.77	-
$\lambda_p = 10$ present work		7.27	6.44
$\lambda_p = 100$ present work		7.61	2.06
$\lambda_p = 1000$ present work		7.67	1.28

compared to the experimental work of (BHAGA, 1976). In this case, only $\lambda_p = 10$ and $\lambda_p = 100$ were simulated. The time history of the bubble Reynolds profile is shown in Fig. (4.20). The terminal Reynolds number when $\lambda_p = 10$ was $Re = 0.0693$, while $\lambda_p = 100$ yielded $Re = 0.0753$. The error in the first case was $\epsilon = 11.12\%$, decreasing in the second case to $\epsilon = 3.42\%$.

The last case investigated in this section was a wobbling bubble with $Eo = 10$ and $M = 9.78e-8$. The same case was simulated with $\lambda_p = 10$ and $\lambda_p = 100$. The density ratio, in this case,

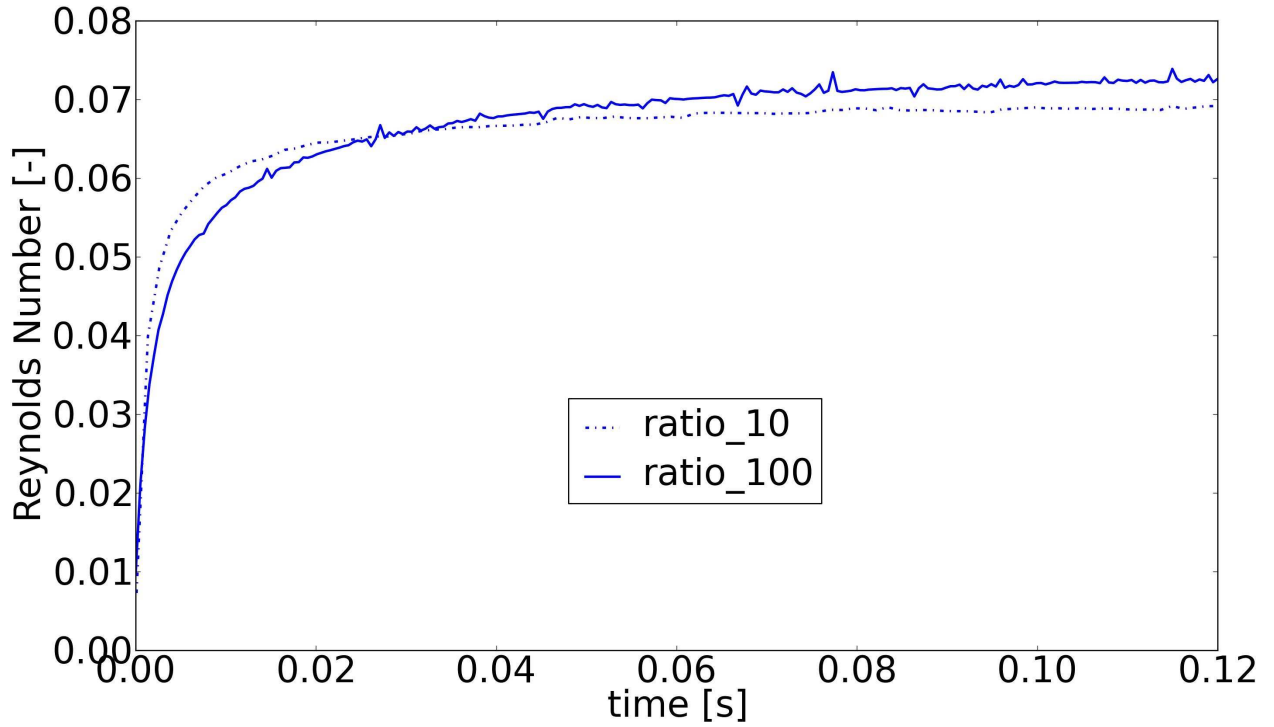


Figure 4.20: Reynolds profile for a spherical rising bubble at $Eo=8.67$ and $M=711$. Experimental terminal $Re = 0.078$ according to Bhaga and Weber (1981).

played a fundamental role on the definition of the bubble zigzag path.

Figure (4.21) shows an overlay of some temporal snapshots of the bubble rising in both situations. Bubbles in green refer to $\lambda_p = 10$ and bubbles in grey refer to $\lambda_p = 100$. The same picture shows, on the right, the path of the bubble centroid in both situations. The straight line represents the centroid of the $\lambda_p = 10$ simulation and the dotted line represents the path of the $\lambda_p = 100$ simulation.

Figure (4.22) shows the streamlines inside and around the bubble when $\lambda_p = 10$. A white noise with intensity 50% was added to the initial velocity field in order to force the wobbling. A toroidal vortex can be seen inside the bubble. Notice that the intensity of the vortex is greater on the right side of the bubble, which has larger curvature. A vortex can also be seen downstream, behind the bubble. The interaction between the inner and outer vortices initiates the oscillating bubble movement. The finest level of the Eulerian grid at the same time instant is shown in Fig. (4.23).